Topological nonrigidity of nonuniform lattices

Stanley S. Chang and Shmuel Weinberger[1]

I. Introduction

We say that an arbitrary manifold $(M, \partial M)$ is *topologically rigid relative to its ends* if it satisfies the following condition. If $(N, \partial N)$ is any other manifold with a compact subset $C \subset N$ for which a proper homotopy equivalence $h\colon (N, \partial N) \to (M, \partial M)$ is a homeomorphism on $\partial N \cup (N \backslash C)$, then there is a compact subset $K \subset N$ and a proper homotopy $h_t\colon (N, \partial N) \to (M, \partial M)$ from h to a homeomorphism such that h_t and h agree on $\partial N \cup (N \backslash K)$ for all $t \in [0,1]$. We say that a manifold M without boundary is *properly rigid* or *absolutely topologically rigid* if we eliminate the requirement that h is a homeomorphism on $\partial N \cup (N \backslash C)$ and agrees with h_t on $\partial N \cup (N \backslash K)$ for all $t \in [0,1]$. Along the lines of the classical Borel conjecture that all closed aspherical manifolds are topologically rigid, Farrell and Jones [FJ] provide the following important theorem.

Theorem: Let $m \geq 5$. Suppose that M^m is an aspherical, complete non-positively curved Riemannian manifold with Riemann curvature tensor R. If the i-th covariant derivative $\nabla^i R$ is bounded for all i (although not necessarily uniformly in i), then M is topologically rigid relative to its ends.

In particular, if G is a linear Lie group, i.e. a virtually connected Lie group admitting a faithful representation $\rho\colon G \to \mathrm{GL}_n(\mathbb{R})$ for some n, then the hypotheses of the theorem are satisfied by the double coset space $\Gamma \backslash G/K$. Here K is a maximal compact subgroup of G and $\Gamma \subset G$ is a torsion-free discrete subgroup. This result shows that topological rigidity extends beyond the usual geometric rigidity theory of Mostow and Margulis, which is ordinarily discussed in the context of *arithmetic manifolds.* These manifolds are double coset spaces $\Gamma \backslash G_{\mathbb{R}}/K$, where G is a semisimple algebraic subgroup of GL_n defined over $\mathbb{Q}$, the subgroup K is maximal compact in the real points $G_{\mathbb{R}}$, and Γ is a torsion-free arithmetic subgroup of its rational points $G_{\mathbb{Q}}$.

In this paper, we will be interested in topological rigidity for arithmetic manifolds, not relative to their ends. If $\Gamma \backslash G/K$ is noncompact, Borel and Serre [BS] construct a well-known compactification $\overline{M}$ of M whose Γ-cover has boundary homotopy equivalent to a countably infinite wedge of $(r-1)$-spheres, where r is the rational rank of G. Recall that, if G is a $\mathbb{Q}$-subgroup of $\mathrm{SL}_n(\mathbb{R})$ and Γ is commensurable with $G_{\mathbb{Z}}$, then the *rational rank* $\mathrm{rank}_{\mathbb{Q}}(\Gamma)$ of G is the dimension of any maximal $\mathbb{Q}$-split torus of G. In fact, certain curvature and rigidity phenomena occur or fail to occur in arithmetic manifolds in accordance with the size of its rational rank. Block and Weinberger [BW] prove that $M = \Gamma \backslash G/K$ admits a metric of positive scalar curvature iff $\mathrm{rank}_{\mathbb{Q}}(\Gamma) \geq 3$, although such positively curved metrics never belong to the same coarse class as the natural metric on M inherited from the Lie group structure of G (see Chang [C]).

While arithmetic spaces are always topologically rigid in the category of continuous coarsely Lipschitz maps, i.e. the bounded structure set $S^{\mathrm{bdd}}(\Gamma \backslash G/K)$ vanishes [CW], the size of their proper structure set is conjecturally determined by its rational rank. When $\mathrm{rank}_{\mathbb{Q}}(\Gamma) \leq 1$, then the above theorem of Farrell and Jones, together with the results of [FH] and of Gromov on the structure of cusps, implies that $\Gamma \backslash G/K$ is indeed rigid in the category of proper maps (note that $\mathrm{rank}_{\mathbb{Q}}(\Gamma) = 0$ implies that $\Gamma \backslash G/K$ is compact by well-known theorems of Borel and Harish-Chandra [BH]). Block

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and Weinberger give a plausible argument in [BW] suggesting that the same phenomenon occurs when $\mathrm{rank}_{\mathbb{Q}}(\Gamma) = 2$. In this paper we demonstrate a sort of converse:

Theorem: Let $M = \Gamma\backslash G/K$ be a noncompact arithmetic manifold for which $\mathrm{rank}_{\mathbb{Q}}(\Gamma) \geq 3$. Then M has a finite-sheeted cover N whose proper structure set $S^p(N)$ is nontrivial, i.e. there is a manifold X with a proper homotopy equivalence $g\colon X \to N$ that is not properly homotopic to a homeomorphism.

This result notably shows that Mostow's rigidity theorem, which holds for noncompact hyperbolic manifolds of finite volume (rational rank 1) but fails spectacularly for general symmetric manifolds, cannot be weakened to provide a proper version of Borel's conjecture for manifolds of noncompact type.

The proof of the above theorem combines a number of well-known but deep results: theorems of Sullivan and Wall from classical surgery theory [Wa], the Borel-Serre compactification of arithmetic manifolds [BS], Kazhdan's property T (see [Z]) and a consequence by Lubotzky [L] of Weisfeiler's strong approximation for linear groups [Wei].

Note: In the final section of this paper, we will extend the theorem to nonarithmetic nonuniform lattices using a geometric generalization of the idea of $\mathbb{Q}$-rank.

II. Group-theoretic background

Weisfeiler's strong approximation result for general linear groups [Wei] states that, if Γ is a Zariski-dense subgroup in an algebraic group G, then Γ is virtually dense in G with respect to the congruence topology, i.e. the closure of Γ is of finite index in $\widehat{G}$. The congruence topology of $\mathrm{SL}_n(\mathbb{Z})$, for example, is the topology for which the groups $\Gamma(m) = \ker(\mathrm{SL}_n(\mathbb{Z}) \to \mathrm{SL}_n(\mathbb{Z}_m))$ serve as a system of neighborhoods of the identity and its completion is $\Pi_p \mathrm{SL}_n(\widehat{\mathbb{Z}}_p)$. The theorem implies, in particular, that a finitely generated linear group is either solvable or has a finite index subgroup with infinitely many different finite simple quotients. Lubotzky uses Weisfeiler's result to prove the following:

Theorem A [L]: Let $\mathbb{F}$ be a field of characteristic different from 2 or 3, and let Γ be a finitely generated infinite subgroup of $\mathrm{GL}_n(\mathbb{F})$. For all $d \in \mathbb{Z}_{\geq 1}$, there is a finite index subgroup of Γ whose index in Γ is divisible by d.

Wehrfritz gave a different proof [Weh] of the above result for $d = 2$ that is also valid in characteristics 2 and 3, which is all we need for our main result. It is worth noting that Theorem A is equivalent to the assertion that, for any prime q, the q-Sylow subgroup of the profinite completion $\widehat{\Gamma}$ of Γ is infinite. In our discussion it will be convenient to use the following strengthening of Theorem A:

Theorem B [L]: Suppose that Γ satisfies the hypotheses of Theorem A and is not solvable-by-finite, i.e. Γ has no solvable subgroup of finite index. Then for every prime q, the q-Sylow subgroup of $\widehat{\Gamma}$ is infinitely generated.

Corollary: Let Γ be any linear group. There is then a normal subgroup $\Gamma'' \triangleleft \Gamma$ of finite even index, and hence Γ contains a subgroup Γ' with a homomorphism onto $\mathbb{Z}_2$.

Proof: Since every subgroup of finite index contains a normal subgroup of finite index, the first remark follows. Using Cauchy's theorem, there is a subgroup of Γ/Γ'' isomorphic to $\mathbb{Z}_2$. Let Γ' be the inverse image of this subgroup.

III. Main Theorem

Proposition: Let G be a semisimple Lie group with trivial center and $\mathrm{rank}_{\mathbb{R}}(G) \geq 2$. Let $M = \Gamma\backslash G/K$ be an arithmetic manifold with Γ an irreducible lattice of G. Then M has a finite cover N for which $H_1(N)$ contains 2-torsion.

For any locally compact group G, recall that G has *Kazhdan property* T if any unitary representation $\pi\colon G \to U(\mathcal{H})$ of G on a Hilbert space $\mathcal{H}$ which almost has invariant vectors actually has nontrivial invariant vectors. Kazhdan [K] proves that, if G is a connected semisimple Lie group with finite center, each of whose factors has real rank at least two, then G, as well as any lattice subgroup of G, has Kazhdan property T. This property stands opposite the condition of amenability in the sense that, if G is amenable, then G has Kazhdan property T iff G is compact. From this result it is easy to show that, if $\phi\colon G \to H$ is a homomorphism where G has Kazhdan property T and H is amenable, then $\overline{\phi(G)}$ is compact.

Proof: The corollary above asserts the existence of a subgroup $\Gamma' \leq \Gamma$ equipped an epimorphism $\phi\colon \Gamma' \to \mathbb{Z}_2$. Let $H = \ker\phi$ so that $\Gamma'/H \cong \mathbb{Z}_2$. Let $J = [\Gamma'\colon \Gamma']$ be the commutator subgroup of Γ'. Observe that both H and J are normal in Γ'. Now consider the quotient homomorphism $\rho\colon \Gamma' \to \Gamma'/J$. First assume that G is simple. By the condition on the real rank of G, it follows that Γ' has property T. Since Γ'/J is amenable, the image $\rho(\Gamma') = \Gamma'/J$ must be compact, and hence the discreteness of Γ, the index $[\Gamma'\colon J]$ is finite. Therefore let N be the cover of M with respect to the subgroup $\Gamma' \leq \Gamma$. Then $\pi_1(N) = \Gamma'$ and $H_1(N) = \Gamma'/J$ contains 2-torsion.

In the case that G is not simple, we use superrigidity of Γ' in place of property T [Ma]. Since a positive Betti number of N would give a homomorphism to S^1 with infinite image (by sending the generator of $\mathbb{Z}$ to an irrational rotation), according to superrigidity G would have to have such a homomorphism. As it does not, all the lattices that we consider have vanishing b_1. Therefore the first integral homology of N is finite. The remainder of the proof proceeds as in the simple case.

Corollary: Let N be given as above. Then the group $H^2(N, \mathbb{Z}_2)$ is nonzero.

Proof: By the above proposition, the homology group $H_1(N)$ contains 2-torsion. We then conclude that $\mathrm{Ext}(H_1(N), \mathbb{Z}_2)$ is nontrivial, since $\mathrm{Ext}(\mathbb{Z}_m, \mathbb{Z}_n) \cong \mathbb{Z}_{(m,n)}$ for any $m, n \in \mathbb{Z}_{\geq 1}$. By the Universal Coefficient Theorem, the map $\mathrm{Ext}(H_1(N), \mathbb{Z}_2) \to H^2(N, \mathbb{Z}_2)$ is injective, so $H^2(N, \mathbb{Z}_2)$ is nontrivial as well.

Theorem 1: Let $M = \Gamma\backslash G/K$ be a noncompact arithmetic manifold whose $\mathbb{Q}$-rank is at least 3. Then M has a finite-sheeted cover N whose proper structure set is nontrivial; i.e. the manifold M is virtually properly rigid.

Proof: Let Γ' be a normal subgroup of Γ of finite even index and let N be the cover of M corresponding to Γ'. Then $\pi_1(N) = \Gamma'$ and $H^2(N, \mathbb{Z}_2)$ is nonzero. As observed by Block and Weinberger [BW], this N can be compactified to a π-π manifold $\overline{N}$ with boundary since the $\mathbb{Q}$-rank is greater than 2. This result follows from the identification of the homotopy type of the Γ-cover of the boundary with a wedge of $(q-1)$-spheres using the Solomon-Tits theorem [BS]. According to Siebenmann's thesis, any manifold that is properly homotopy equivalent to M will have the same property. Using the h-cobordism theorem, any such manifold has a unique compactification so that the extension of the proper homotopy equivalence to the compactification is a simple homotopy equivalence. We can then identify $S(\overline{N})$ with the proper structure set $S^p(N)$. By the π-π theorem of Wall [Wa], the structure set $S(\overline{N})$ of $\overline{N}$ is isomorphic to $[\overline{N}, F/Top]$. Since $F/Top = K(\mathbb{Z}_2, 2) \times K(\mathbb{Z}_2, 6) \times K(\mathbb{Z}_2, 10) \times \cdots \times Z$ for some space Z [MM], we then have

$$\begin{aligned} S^p(N) &= S(\overline{N}) = [\overline{N}, F/Top] = [N, F/Top] \\ &= [N, K(\mathbb{Z}_2, 2) \times K(\mathbb{Z}_2, 6) \times K(\mathbb{Z}_2, 10) \times \cdots \times Z] \\ &= [N, K(\mathbb{Z}_2, 2)] \times [N, K(\mathbb{Z}_2, 6)] \times \cdots \times [N, Z] \end{aligned}$$

which by the previous corollary is nontrivial since $[N, K(\mathbb{Z}_2, 2)] = H^2(N, \mathbb{Z}_2)$.

Theorem 2: Let $M = \Gamma\backslash G/K$ be an arithmetic manifold with Γ irreducible. If $\mathrm{rank}_{\mathbb{Q}}\Gamma \geq 3$, then M has finite-sheeted covers N whose proper structures are arbitrarily large.

Proof: By the proof of Lubotzky's Theorem B the profinite completion $\widehat{\Gamma}$ of Γ contains an infinitely generated elementary abelian 2-group $\mathbb{Z}_2^\infty$. If M is a fixed positive integer, there is then a finite quotient Γ/I of Γ containing $\mathbb{Z}_2^M$. Let Γ' be the inverse image of this $\mathbb{Z}_2^M$ under the projection map $\Gamma \to \Gamma/I$. If $L/I = [\Gamma'/I, \Gamma'/I]$ is the commutator subgroup of Γ'/I, then the abelianization of Γ'/I is isomorphic to Γ'/L, which is a 2-group with the property that $\mathrm{Ext}(\Gamma'/L, \mathbb{Z}_2)$ has at least 2^M elements. Let $J = [\Gamma', \Gamma']$ and consider the short exact sequence of abelian groups given by $0 \to L/J \to \Gamma'/J \to \Gamma'/L \to 0$. Therefore we have the exact sequence $0 \to \mathrm{Hom}(\Gamma'/L, \mathbb{Z}_2) \to \mathrm{Hom}(\Gamma'/J, \mathbb{Z}_2) \to \mathrm{Hom}(L/J, \mathbb{Z}_2)$. Since all quotients here are finite abelian (superrigidity implies that Γ'/J is finite), we have

$$|\mathrm{Ext}(\Gamma'/J, \mathbb{Z}_2)| = |\mathrm{Hom}(\Gamma'/J, \mathbb{Z}_2)| \geq |\mathrm{Hom}(\Gamma'/L, \mathbb{Z}_2)| = |\mathrm{Ext}(\Gamma'/L, \mathbb{Z})| \geq 2^M.$$

If N is the finite cover of M corresponding to the subgroup $\Gamma' \leq \Gamma = \pi_1(M)$, we can conclude as in the above corollary that $H^2(N, \mathbb{Z}_2)$ is arbitrarily large. Therefore the proper structure set $S^p(N)$ is also arbitrarily large.

Remark: In fact, if the $\mathbb{R}$-rank is large enough and $\mathrm{rank}_{\mathbb{Q}}(\Gamma) > 2$, then one can construct infinite structure sets with nontrivial elements detected by Pontrjagin classes, e.g. for $\mathrm{SL}_n(\mathbb{Z})$ for n sufficiently large, using Borel's calculations [B]. Unlike the elements constructed here, these elements do not die on passage to further finite-sheeted covers. Note that, for a product of three punctured surfaces, the proper rigidity conjecture is always false for any cover, but is virtually true, in that any counterexample dies on passing to another finite cover.

IV. Coarse Volume Growth and Nonarithmetic Lattices

The results in the previous section can be generalized to locally symmetric spaces $M = \Gamma\backslash G/K$ for which we eliminate all irreducibility or arithmeticity requirements on the subgroup Γ.

Definition: Let M be a metric space and let $p \in M$. For any $R > 0$ define

$$\mathrm{cov}_1(B(p,R)) = \inf_k \left\{ k\colon B(p,R) \subset \bigcup_{i=1}^k B(p_i, 1) \text{ for some } p_1, \ldots, p_k \in M \right\}.$$

We denote by $\mathrm{cvg}(M)$, the *coarse volume growth* of M, to be the quantity

$$\mathrm{cvg}(M) = \lim_{R\to\infty} \frac{\log \mathrm{cov}_1(B(p,R))}{\log R}.$$

It is clear that the coarse volume growth of M is independent of the basepoint p.

Remark 1: The quantity $\mathrm{cov}_1(B(p,R))$ generalizes the notion of the growth rate of groups, given by the function $f_G(n) = |B(x,n)|$, where $B(x,n)$ denotes the ball of radius n about a fixed vertex x in the Cayley graph of G with the usual word length metric. Note, for instance, that $\mathrm{cvg}(\mathbb{R}^n) = n$ when $\mathbb{R}^n$ is endowed with the usual Euclidean metric and $\mathrm{cvg}(M) = 0$ when M is bounded. If P is an n-dimensional simplicial complex and $M = cP$ is the open cone on P, then $\mathrm{cvg}(M) = n+1$. Coarse volume growth enjoys many properties exhibited by rational rank. For example, it is additive over products; i.e. if M_1 and M_2 are metric spaces, then $\mathrm{cvg}(M_1 \times M_2) = \mathrm{cvg}(M_1) + \mathrm{cvg}(M_2)$ when $M_1 \times M_2$ is given the usual product metric.

Remark 2: Technically the notion of rational rank applies only when Γ is arithmetic, although as mentioned in [Mo] one can extend the definition to all lattices using the Margulis arithmeticity theorem. In particular, if Γ is a lattice of a semisimple Lie group G, then up to isogeny and modulo the maximal compact factor of G we can write $G = G_1 \times \cdots \times G_r$ so that $\Gamma_i = \Gamma \cap G_i$ is an irreducible lattice in G_i for all i. One can then define $\mathrm{Rank}_{\mathbb{Q}}(\Gamma) = \mathrm{Rank}_{\mathbb{Q}}(\Gamma_1) + \cdots + \mathrm{Rank}_{\mathbb{Q}}(\Gamma_r)$, where

$$\mathrm{Rank}_{\mathbb{Q}}(\Gamma_i) = \begin{cases} 0 & \text{if } \mathrm{rank}_{\mathbb{R}}(G_i) = 0, \\ 1 & \text{if } \mathrm{rank}_{\mathbb{R}}(G_i) = 1, \\ \mathrm{rank}_{\mathbb{Q}}(\Gamma_i) & \text{otherwise.} \end{cases}$$

This apparently forced generalization of rational rank is actually consistent with the coarse volume growth in the context of locally symmetric spaces. Although it is a more extended concept than is required in this paper, we will continue to use this latter form of volume growth as a more natural large-scale geometric measure of general metric spaces.

Remark 3: Certainly coarse volume growth is a coarse invariant. By Ji and MacPherson [JM], if M is an arithmetic manifold, then M is coarsely equivalent to the metric cone over the Tits complex $\Delta(\Gamma\backslash G/K)$ of dimension $\mathrm{rank}_{\mathbb{Q}}(\Gamma) - 1$. We therefore have the following.

Proposition: If the manifold $M = \Gamma\backslash G/K$ is arithmetic, then the coarse volume growth of M is the dimension of the tangent cone of M, i.e. $\mathrm{cvg}(M) = \mathrm{rank}_{\mathbb{Q}}(\Gamma)$. In general, it agrees with the extension mentioned in Remark 2.

Proposition: Let G be a semisimple Lie group and let the locally symmetric manifold $M = \Gamma\backslash G/K$ be endowed with the natural metric inherited from G. Here Γ is a lattice of G which is not necessarily irreducible or arithmetic. If the coarse volume growth of M is at least 3, then M is a π-π manifold.

Proof: As mentioned above, if Γ is irreducible, it follows from [BS] and the structure of cusps [G2] that then the universal cover $\widetilde{\partial}(M)$ of the boundary of M is homotopy equivalent to a wedge of spheres of dimension $\mathrm{cvg}(M) - 1$. The general case is proved using the observation that, of $\mathrm{cvg}(\Gamma\backslash G/K) = q$ and $\mathrm{cvg}(\Gamma'\backslash G'/K') = r$, then homotopically we have

$$\widetilde{\partial}(\Gamma\backslash G/K \times \Gamma'\backslash G'/K') = \widetilde{\partial}(\Gamma\backslash G/K) * \widetilde{\partial}(\Gamma'\backslash G'/K') = \bigvee S^{q-1} * \bigvee S^{r-1} = \bigvee S^{q+r-1}.$$

Theorem: Let G be a semisimple Lie group and let Γ be a lattice of G (with no assumptions on arithmeticity or irreducibility). If $M = \Gamma\backslash G/K$ and $\mathrm{cvg}(M) \geq 3$ then there is a finite-sheeted cover N of M which is not properly rigid.

Proof: Given the condition on the coarse volume growth and the proposition above, we know that M can be compactified to a π-π manifold, so that $[M, F/Top] = S^p(M)$. If Γ is irreducible, then M is arithmetic by Margulis, and the proof can be completed as before. If Γ is reducible, then write $M = M' \times M''$, where $M' = \Gamma' \backslash G'/K'$ and $M'' = \Gamma'' \backslash G''/K''$. Note that it is sufficient to prove that $H^2(N, \mathbb{Z}_2) \neq 0$ for some cover N of M. et N' and N'' be finite-sheeted covers of M' and M'' equipped with surjections $\phi_1 \colon \pi_1(N') \to \mathbb{Z}_2$ and $\phi_2 \colon \pi_1(N'') \to \mathbb{Z}_2$.

Case 1: Suppose that both ϕ_1 and ϕ_2 can be chosen so that they factor through $\mathbb{Z}$. Then $\mathbb{Z}$ is a summand of both $H_1(N')$ and $H_1(N'')$, so that $\mathbb{Z}_2$ is a summand of both $H^1(N', \mathbb{Z}_2)$ and $H^1(N'', \mathbb{Z}_2)$. If $N = N' \times N''$, then $H^2(N, \mathbb{Z}_2)$ is nonzero.

Case 2: Suppose that there is no surjection $\phi_1 \colon \pi_1(N') \to \mathbb{Z}_2$ that factors through $\mathbb{Z}$. Let J be the commutator subgroup of $\pi_1(N')$. If $H_1(N')$ has no 2-torsion, then $H_1(N') = \pi_1(N')/J$ must be infinite since $[\pi_1(N') \colon J] = [\pi_1(N') \colon \ker \phi][\ker \phi \colon J] = 2\,[\ker \phi \colon J]$. Hence $\pi_1(N')/J \cong \mathbb{Z} \oplus R$ for some abelian group R. Therefore the composite $\pi_1(N') \to \pi_1(N')/J \to \mathbb{Z} \oplus R \to \mathbb{Z}$ is a surjection, yielding a contradiction. If $N = N' \times M''$, then $H_1(N) = H_0(N') \otimes H_1(M'') \oplus H_1(N') \otimes H_0(M'') = H_1(M'') \oplus H_1(N')$ has 2-torsion, and so $H^2(N, \mathbb{Z}_2)$ is nonzero by the Universal Coefficient Theorem.

Remark: We (and Jonathan Block) note that the existence of uniformly positive scalar curvature metrics on $\Gamma \backslash G/K$ when Γ is irreducible and $\operatorname{rank}_{\mathbb{Q}}(\Gamma) \geq 3$ established in [BW] can be proved for any locally symmetric space M with $\operatorname{cvg}(M) \geq 3$ by the above method; moreover, if $\operatorname{cvg}(M) \leq 2$, then M has no complete metric of positive scalar curvature.

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References

[B] A. Borel, *Stable real cohomology of arithmetic groups*, Ann. Sci. École Norm. Sup. (4) 7 (1974), 235–272 (1975).

[BS] A. Borel and J.P. Serre, *Corners and arithmetic groups.* Comment. Math. Helv. 48 (1973), 436–491.

[BW] J. Block and S. Weinberger, *Arithmetic manifolds of positive scalar curvature.* J. Diff. Geo. 52 (1999), no. 2, 375–406.

[FJ] F.T. Farrell and L. Jones, *Rigidity for aspherical manifolds with* $\pi_1 \subset \mathrm{GL}_m(\mathbb{R})$. Asian J. Math. 2 (1998), no. 2, 215–262.

[G1] M. Gromov, *Groups of polynomial growth and expanding maps*, Inst. Hautes Études Sci. Publ. Math. no. 53 (1981), 53–73.

[G2] M. Gromov, *Almost flat manifolds,* J. Differential Geom. 13 (1978), no. 2, 231-241.

[JM] L. Ji and R. MacPherson, *Geometry of compactifications of locally symmetric spaces*, Ann. Inst. Fourier (Grenoble) 52 (2002), no. 2, 457–559.

[L] A. Lubotzky, *On finite index subgroups of linear groups.* Bull. London Math. Soc. 19 (1987), no. 4, 325–328.

[MM] I. Madsen and R.J. Milgram, *The classifying spaces for surgery and cobordism of manifolds*, Annals of Mathematics Studies, 92. Princeton University Press, Princeton, N.J.; University of Tokyo Press, Tokyo, 1979.

[Ma] G. Margulis, *Discrete subgroups of semisimple Lie groups*, Ergebnisse der Mathematik und ihrer Grenzgebiete (3) [Results in Mathematics and Related Areas (3)], 17. Springer-Verlag, Berlin, 1991.

[Mo] D. Morris, *Introduction to arithmetic groups*, preprint, available at http://people.uleth.ca /~dave.morris/lectures/ArithGrps/Morris-ArithGrps-Feb03.pdf

[Wa] C.T.C. Wall, *Surgery on compact manifolds*, second edition. American Mathematical Society, Providence, RI, 1999.

[We] B. Weisfeiler, *Strong approximation for Zariski-dense subgroups of semisimple algebraic groups.* Ann. of Math. (2) 120 (1984), no. 2, 271–315.

[Z] R. Zimmer, *Ergodic theory and semisimple groups.* Monographs in Mathematics 81. Birkhäuser Verlag, Basel-Boston, Mass., 1984. x+209 pp.

Department of Mathematics, 106 Central Street, Wellesley College, Wellesley MA 02481
sschang@palmer.wellesley.edu

Department of Mathematics, 5734 University Ave, University of Chicago, Chicago IL 60637
shmuel@math.uchicago.edu