HALLOWEEN FAKE TEST

- Complete the following fake problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes fake exam. No calculators or other electronic aids will be permitted.
- In order to receive full fake credit, please show all of your work and justify your fake answers. You do not need to simplify your fake answers unless specifically fake instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this fake exam. Do not unstaple or detach pages from this fake exam.
- Please sign the following:

  "On my honor, I have neither given nor received any aid on this fake examination. I have furthermore abided by all other aspects of the honor code with respect to this fake examination."

  Signature: ________________________________

The following boxes are strictly for fake grading purposes. Please do not mark.

1	10^6 pts	
2	0 pts	
3	$e^{i\pi}$ pts	
4	∞ pts	
5	10^{-10} pts	
6	π pts	
7	π^π pts	
8	λ pts	
Total	100 pts	

(1) If A is invertible and λ is an eigenvalue of A, show λ^{-1} is an eigenvalue of A^{-1}.

(2) Suppose that λ is an eigenvalue for A. Show that λ^2 is an eigenvalue for A^2.

(3) Prove that $\det(A - \lambda I_n) = 0$ if and only if $\det(\lambda I_n - A) = 0$. (This means you can find eigenvalues either way.)

(4) Suppose that $V \subseteq \mathbb{R}^n$ is a *proper* subspace of $\mathbb{R}^n$ (i.e., $0 < \dim V < \dim \mathbb{R}^n$). Prove that 0 and 1 are eigenvalues of proj_V.

(5) Suppose that A is a square matrix whose columns form an orthonormal basis of $\mathbb{R}^n$. Prove that $\det(A) = \pm 1$. (Hint: You might want to use the following two facts. First, for any square matrix B it is a fact that $\det(B) = \det(B^T)$. Second, for square matrices B and C, $\det(BC) = \det(B)\det(C)$.)

(6) Carefully show that for any square matrix A,

$$\det(A - \lambda I_n) = \det(A^T - \lambda I_n).$$

(You can use the fact that for any matrix B, $\det(B) = \det(B^T)$.) Explain what this means in terms of the eigenvalues of A and A^T.

(7) Suppose that A is a matrix whose columns for an orthonormal basis for $\mathbb{R}^n$. Prove that $\overrightarrow{u} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$ is an eigenvector of A^T. What is its corresponding eigenvalue? Use this fact and the previous problem to produce an eigenvalue of A.

(8) For some $n \times n$ matrix A, Let E_λ be the eigenspace associated to λ; that is,

$$E_\lambda = \{\overrightarrow{v} \in \mathbb{R}^n : A\overrightarrow{v} = \lambda\overrightarrow{v}\}.$$

Show that E_λ is a subspace of $\mathbb{R}^n$.