Consider the following elements of S_7:

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 4 & 1 & 5 & 6 & 7 \end{bmatrix}, \quad \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 5 & 2 & 4 & 7 & 1 & 3 \end{bmatrix}, \quad \gamma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 4 & 7 & 1 & 2 & 3 & 6 \end{bmatrix}.$$

(1) Write α, β and γ in disjoint cycle notation.
(2) Calculate $\alpha\beta, \beta\alpha, \beta^{-1}, \gamma\alpha^{-1}\beta, \gamma\beta\gamma^{-1}$. Express your answers in disjoint cycle notation.
(3) Calculate the order of α, β and γ.
(4) Compute $\langle\alpha\rangle$ and $\langle\gamma\rangle$.
(5) Find all generators for $\langle\gamma^8\rangle$.
(6) What is the β-conjugate of $\langle\alpha\rangle$?
(7) List all of the subgroups of $\langle\gamma\rangle$ and all the subgroups of $\langle\beta\rangle$. Does $\langle\alpha\rangle$ have any interesting subgroups?
(8) Find 3 elements in S_7 that have order 3. Can S_7 be cyclic? Explain.
(9) Write α, β and γ as products of transpositions. Is each even or odd?
(10) Is $\beta \in Z(S_7)$?
(11) Find an element in $C(\alpha)$ that is not in $\langle\alpha\rangle$.