# MATH 51: MATLAB HOMEWORK 2

1. INTRODUCTION

In class you have learned how Google uses linear algebra to rank webpages. In this assignment you will be implementing various approximations of the Google pagerank. All the terminology we use can be found in the Google handout, which can be found at http://math.stanford.edu/ mark/math51-05/PageRank.pdf.

2. EXERCISES

Suppose your universe of webpages is given by the following schematic, in which each webpage is labeled by a numbered circle, and where an arrow from circle i to circle j indicates that page i has a link to page j:
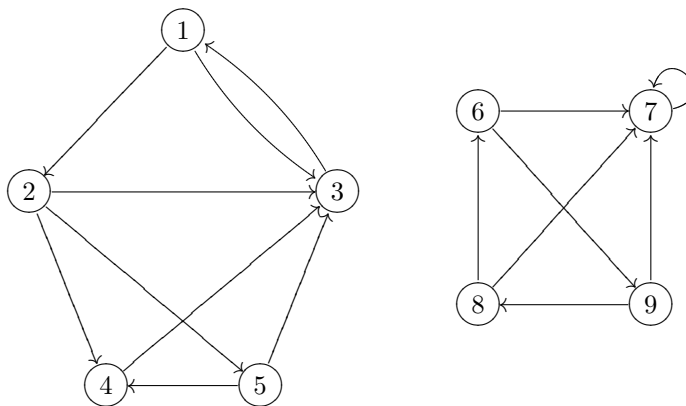
(1) Write out the "linking matrix" A, the 9×9 matrix whose ij entry is

$$a_{ij} = \begin{cases} 1 & \text{if page } j \text{ has a link to page } i, \text{ or if page } j \text{ is a dead end} \\ 0 & \text{if not} \end{cases}$$

(2) Compute the "ranking vector" $\mathbf{v}$, given by

$$\mathbf{v} = A\mathbf{u},$$

where $\mathbf{u} \in \mathbf{R}^9$ is the column vector with all entries 1.

(3) Use $\mathbf{v}$ to rank the nine pages, in order from highest ranked to lowest ranked (see §1, "The Very Simple Method," of the Google handout).

(4) For each $1 \leq j \leq 9$, compute $n_j = \sum a_{ij}$ (here a_{ij} are the entries of the matrix A above). Use this to write down the improved "linking matrix" P, the 9×9 matrix whose ij entry is

$$p_{ij} = \frac{a_{ij}}{n_j}.$$

(5) Verify that your matrix P is a Markov matrix by showing the entries in each column sum to 1 (you should show at least 2 of your computations).

(6) Compute the "ranking vector" $\mathbf{v}$, given by

$$\mathbf{v} = P\mathbf{u},$$

where $\mathbf{u} \in \mathbf{R}^9$ is the column vector with all entries 1, and use $\mathbf{v}$ to rank the nine pages in order from highest ranked to lowest ranked (see §2, "First Improvement," of the Google handout).

(7) Find two linearly independent eigenvectors with eigenvalue 1 for the matrix P above. Use each of these vectors to rank the nine pages in order from highest ranked to lowest ranked (see §3, "A Further Improvement," of the Google handout).

(8) With the parameter $r = 0.85$ and the matrix T whose ij entry is $\frac{1}{9}$, write down the matrix $Q = rP + (1 - r)T$. Verify that Q is still a Markov matrix by showing the entries in each column sum to 1 (you should show at least 2 of your computations).

(9) Find an eigenvector with eigenvalue 1 for the matrix Q (notice that it is unique up to scaling). Use this vector to rank the nine pages in order from the highest ranked to the lowest ranked (see §4, "Final Improvement," of the Google handout).

(10) Repeat exercises 1-9 above, with the following universe of webpages:
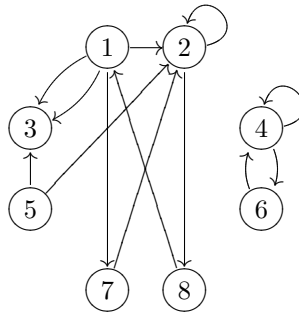
(Notice that now there are 8 webpages, so your matrices A, P, and Q will be 8×8, etc. Also note that for exercise 7, there will not be two linearly independent eigenvectors with eigenvalue 1; just find one.)