COURSE NOTES - 03/04/05

1. Recap

Last class period we were introduced to the Extreme Value Theorem and its applications. Along the way we revisited local (a.k.a. relative) maxima and minima and met absolute maxima and minima of a function. We also defined what a critical number is for a function $f(x)$. One we had this terminology, we stated the Extreme Value Theorem and used it to develop a 3 step method for finding absolute maxima and minima of a function $f(x)$ on a closed interval.

2. Homework problems

We discussed a handful of homework problems today. Solutions for these and all homework exercises are posted on the course webpage, as per usual. (These will be posted immediately after class on Monday)

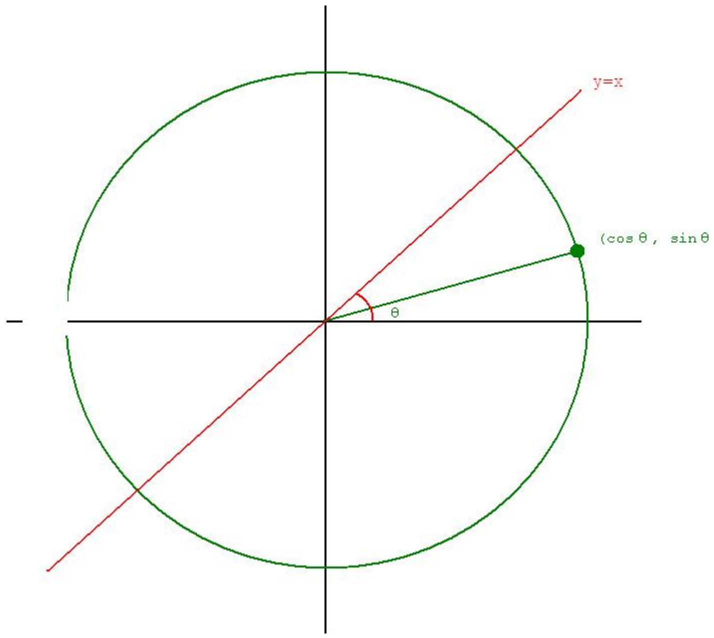
3. Another practice problem

Example. Find the absolute maximum and minimum values of $f(x) = \sin(x) + \cos(x)$ on the interval $[0, \pi/3]$.

Solution. We will use the 3 step technique we introduced in the last class to find the absolute max and min of this function.

- Since $f(x)$ is the sum of $\sin(x)$ and $\cos(x)$, each continuous everywhere, we know $f(x)$ is continuous on the closed interval $[0, \pi/3]$.
- To find critical points of $f(x)$, we calculate $f'(x) = \cos(x) - \sin(x)$. Now $f'(x)$ is defined everywhere (since $\cos(x)$ and $\sin(x)$ are defined everywhere), so critical points for f are solutions to $f'(x) = 0$. But $f'(x) = 0$ if and only if $\cos(x) - \sin(x) = 0$, which is equivalent to $\cos(x) = \sin(x)$. For what values of x does $\cos(x) = \sin(x)$? This question tests our knowledge of trigonometry, so in many ways it is tangential to the course (pun, sadly, intended). However, we need an answer.

  Trig wiz kids know that $\sin(x) = \cos(x)$ for values $\frac{\pi}{4} + n\pi$ where n is any integer. How do mere mortals arrive at such an answer? Recall that the x-coordinate of a point on the unit circle represents the cosine of the corresponding angle, and the y-coordinate of a point on the unit circle represents the sine of the corresponding angle. Hence $\cos(x) = \sin(x)$ whenever these coordinates agree, which is to say when such a point lies on the line $y = x$. If we sketch the unit circle and the line $y = x$, we see the only point on the unit circle with angle measure in $[0, \pi/3]$ is $\pi/4$. Hence, $\pi/4$ is our critical number.



- Now we exam the values of $f(x)$ at the endpoints of our interval and any critical numbers we found.

x	$f(x)$
0	$\sin(0) + \cos(0) = 0 + 1 = 1$
$\pi/4$	$\sin(\frac{\pi}{4}) + \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$
$\pi/3$	$\sin(\frac{\pi}{3}) + \cos(\frac{\pi}{3}) = \frac{\sqrt{3}}{2} + \frac{1}{2} = \dfrac{\sqrt{3}+1}{2}$

It is clear from the chart that 0 is the absolute minimum. That $\pi/4$ gives the absolute maximum requires that we can show $\sqrt{2} > \dfrac{\sqrt{3}+1}{2}$. We could do this easily with a calculator, though sadly I don't think we have the skills required to answer this without a calculator. Of course on a test or a quiz, you will never need a calculator to compare values.

$\square$