Midterm 1

- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.

- In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so.

- If you need extra room, use the back sides of each page. Extra scratch paper has also been provided at the end of the exam. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

- Please sign the following:

  "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

  Signature: _____

  Name: _____

The following boxes are strictly for grading purposes. Please do not mark.

1	15 pts	
2	15 pts	
3	14 pts	
4	14 pts	
5	14 pts	
6	7 pts	
7	7 pts	
8	14 pts	
Total	100 pts	

1. (15 pts) Complete each of the following sentences clearly and concisely.

   (a) A function f is said to be continuous at a point a if

   (b) The statement of the Intermediate Value Theorem is

   (c) For a function f and a point a in the range of f, $f^{-1}(a)$ is defined to be

2. (15 pts) Determine whether each statement is true or false for arbitrary functions $f(x)$ and $g(x)$. **If the statement is true, cite your reasoning. If it is false, provide a counterexample.** You will receive little or no credit for an answer that does not have justification.

(a) If f is an even function whose domain is all real numbers, then $f^{-1}(x)$ is not a function.

(b) If $\lim_{x \to a} f(x) = 0$, then $\lim_{x \to a} (f \cdot g)(x) = 0$.

(c) If $f(a)$ does not exist, then $\lim_{x \to a} f(x)$ does not exist.

3. (14 pts) For each of the following conditions, provide a specific function $f(x)$ which satisfies the given condition. You may express your function either with an explicit formula or by a graph.

(a) $f(x)$ is defined on all of $[a, b]$. It satisfies $f(a) < 0$ and $f(b) > 0$, but there is no $c \in (a, b)$ with $f(c) = 0$.

(b) $f^{-1}(x)$ is not a function.

4. (14 pts) Compute the following limits, if they exist. If a limit diverges, you may write "diverges."

(a) $\lim_{x \to 1} \dfrac{2x - \sqrt{x^2 + 3}}{x^2 - 3x + 2}$

(b) $\lim_{t \to 1} \dfrac{2t^2 - 2}{3t^2 - t - 2}$

5. (14 pts) The Seabreeze division of Globo-tech manufactures consumer-grade brine. Their manufacturing processes are so refined that for any positive real number x, they can produce exactly x kiloliters of brine. The cost of producing these x kiloliters of brine is given in dollars by the function $C(x) = 10 + \sqrt{x}$. Market analysis shows that the total revenue from selling x kiloliters of brine is given in dollars by the function $R(x) = 20 \log_2(x + 1)$.

(a) Evaluate $C(0)$, $R(0)$, $C(1)$, and $R(1)$. (Your solutions should be simplified as much as possible)

(b) Is there a value $x \in (0, 1)$ so that $R(x) = C(x)$?

6. (7 pts)

   (a) Find functions $g_1(x)$, $g_2(x)$, and $g_3(x)$ such that $\sin\left(e^{\sqrt{x}}\right) = (g_1 \circ g_2 \circ g_3)(x)$.

   (b) What function is $(g_3 \circ g_2 \circ g_1)(x)$?

7. (7 pts) Compute $f^{-1}(x)$, where $f(x) = \dfrac{3x + 2}{x + 1}$.

8. (14 pts) Let $f(x) = \dfrac{3x + 2}{x + 1}$.

(a) Write $f'(2)$ as a limit.

(b) Compute the limit in part (a) above.

(c) Use your result from (b) to write the equation for the tangent line to $f(x)$ at the point $(2, \frac{8}{3})$.

Scratch Paper

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