Midterm 2

- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so.
- If you need extra room, use the back sides of each page. Extra scratch paper has also been provided at the end of the exam. Do not unstaple or detach pages from this exam.
- Please sign the following:

"On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ______________________________

Name: ______________________________

The following boxes are strictly for grading purposes. Please do not mark.

1	10 pts	
2	15 pts	
3	15 pts	
4	15 pts	
5	15 pts	
6	10 pts	
7	20 pts	
B	5 pts	
Total	100 pts	

1. (10 pts) Complete each of the following sentences clearly and concisely.

   (a) When $f(x)$ is a position function, acceleration is given by the ____________ derivative of f and velocity is given by the ____________ derivative of f.

   (b) Local maxima and minima of a function f occur at points a where the derivative of f

2. (15 pts) Determine whether each statement is true or false for an arbitrary function $f(x)$. **If the statement is true, cite your reasoning. If it is false, provide a specific counterexample.** You will receive little or no credit for an answer that does not have appropriate justification.

(a) If $f'(a) = 0$ and $f''(a) = 0$, then f has neither a local maximum nor a local minimum at a.

(b) If f is continuous at a point a, then f is differentiable at a.

(c) If $f'(x) > 0$ on an interval (a, b), then $f(x)$ is concave up on (a, b).

3. (15 pts) In class we listed three ways a function could fail to be differentiable at a point. List all three and give an example of a function f defined on $(0, 10)$ which displays all of these failures of differentiability.

4. (15 pts)

(a) Use the definition of the derivative as a limit to compute

- $\dfrac{d}{dx}\left[x^3\right]$

- $\dfrac{d}{dx}\left[x\right]$

(b) Use your results from part (a) together with the <u>chain rule</u> and the fact that $(\sqrt[3]{x})^3 = x$ to compute $\dfrac{d}{dx}\left[\sqrt[3]{x}\right]$. (Note: In this problem, you may not use the power rule. Only your solutions from part (a) together with the chain rule).

(c) Use your results from part (a) together with the <u>product rule</u> and the fact that $\sqrt[3]{x} \cdot \sqrt[3]{x} \cdot \sqrt[3]{x} = x$ to compute $\dfrac{d}{dx}\left[\sqrt[3]{x}\right]$. (Note: In this problem, you may not use the power rule. Only your solutions from part (a) together with the product rule).

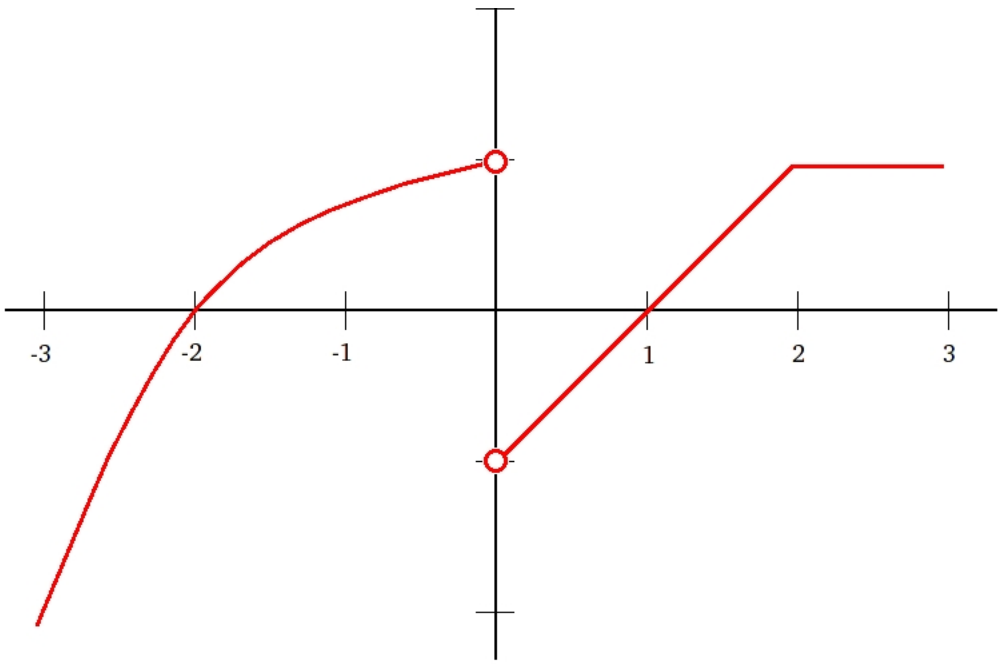
5. (15 pts) A certain function $y = f(x)$ satisfies the following equation: $y^3 + 1 = x^2$.

   (a) Use implicit differentiation to calculate $\dfrac{dy}{dx} = f'(x)$.

   (b) Use your result from part (a) to find the equation of the tangent line of the graph of $y = f(x)$ at the point $(3, 2)$.

   (c) Approximate the value of $f(4)$.

6. (10 pts) For a particular function $f(x)$, we know only that it's derivative $f'(x)$ has the graph



Based on this graph, state the local maxima and minima of the function $f(x)$. Make sure you provide justification for your answer.

7. (20 pts)

(a) $\dfrac{d}{dx}\left[\dfrac{\arcsin(x)}{\sqrt[4]{x^{11}}+\cos(x)}\right]$

(b) $\dfrac{d}{dx}\left[\log(e^{x^3+x+1})\right]$

(c) $\dfrac{d}{dx}\left[2^{\sin(x)\sqrt[3]{x}}\right]$

(d) Calculate $\dfrac{dy}{dx}$ for y satisfying $\tan(y-x)=\sec(x-y)$.

Bonus (5pts) Is it true that if $f'(a)$ and $g'(a)$ are both undefined then $(f+g)'(a)$ is undefined? If so, give a complete justification. If not, provide a specific counterexample. Credit will only be given for complete, correct answers.

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