Midterm 2
(Practice)

- Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- Please sign the following:

  "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

  Signature: ______________________________

The following boxes are strictly for grading purposes. Please do not mark.

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1. Complete each of the following sentences.

   (a) For a function $f(x)$, the derivative $f'(x)$ is defined to be the limit

   (b) The product rule states

   (c) For a function $f(x)$, the second derivative is defined to be

2. Determine whether each statement is true or false for an arbitrary function $f(x)$. **If the statement is true, cite your reasoning. If it is false, provide a counterexample.**

   (a) If $f''(a)$ is not positive, then f cannot have a minimum at a.

   (b) If $f'(a) = 0$, then f has a maximum or a minimum at a.

   (c) If f is differentiable at a point a, then f is continuous at a.

3. For each of the following conditions, provide an example of a function $f(x)$ which satisfies the given condition. Unless otherwise indicated, you may express your function either with an explicit formula or by a graph.

(a)
- $f(0) = 0$,
- $f'(0) > 0$, and
- $f''(0) < 0$.

(b) $f'(0) = 0$ and $f'(1) = 0$, yet f has neither a maximum nor a minimum at either 0 or 1.

4. (a) Compute $\frac{d}{dx}\left[f(x)\,(g(x))^{-1}\right]$ with the product and chain rules (not the quotient rule) to verify

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

(b) Use the fact that $(f \circ f^{-1})(x) = x$, together with the chain rule, to compute $\dfrac{d}{dx}\left[f^{-1}(x)\right]$ in terms of $f'(x)$ and $f^{-1}(x)$. (Recall that $f^{-1}(x)$ is the inverse of $f(x)$, and **not** $\frac{1}{f(x)}$).

(Hint: Take the derivative of each side of the equation $x = (f \circ f^{-1})(x)$ and solve).

5. For a certain function $f(x)$, we know only that $f(9) = 5$ and that $f'(x) = \log_3(x)$.

   (a) Use the technique of linearization to approximate $f(11)$.

   (b) Compute $f''(x)$.

6. A certain function $g(x)$ satisfies $g'(x) = e^{-x^2+1}$.

   (a) Find all local maxima and minima of $g(x)$. (For any maximal/minimal value a, be sure you state—and justify—whether a is a maximum or a minimum of $g(x)$).

   (b) Find all maxima and minima of $g'(x)$. (For any maximal/minimal value a, be sure you state—and justify—whether a is a maximum or a minimum of $g'(x)$).

7. Compute the following derivatives. You may use any techniques or results we discussed in class.

   (a) $\dfrac{d}{dx}\left[x\sqrt{x^2+1}\right]$

   (b) $\dfrac{d}{dx}\left[\sin(e^x)\right]$

(c) $\dfrac{d}{dx}\left[\dfrac{\sin(x)+\cos(x)}{\sin(x)-\cos(x)}\right]$

(d) $\dfrac{d}{dx}\left[\sqrt[3]{\tan(x)}\right]$