THE DERIVATIVE AT A POINT

1. Announcements

- Solutions to Homework 3 will be posted by the end of the day. I won't be able to get them back to you before the test.
- We have a review session tomorrow (Thursday) starting at 6:45. I'll bring lots of pizza and beverages, so come hungry (and thirsty).
- Ben's office hours for the remainder of the week have been cancelled in lieu of his bonus office hours on Tuesday and the review session Thursday. If that's a problem for you, send me a note and we'll get something set up.
- I'll have the tests on desks in the room at 8:55. If you want 5 minutes to read the test before you begin at 9, come then. The test will begin at 9 sharp and will go until 9:50 (also sharp).

2. Derivatives

2.1. The definition of the derivative.

I should have said this last time in class, but didn't have time to mention it.

Definition. For a function $f(x)$, the derivative of $f(x)$ at a, written $f'(a)$, is defined to be

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a},$$

if it exists. If this limit does not exist, we say that $f(x)$ fails to be differentiable at a or that $f'(a)$ is undefined. The derivative $f'(a)$ represents the slope of the line tangent to f at a. From the comments above, we see that we can also evaluate $f'(a)$ by computing a seemingly different limit:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

2.2. A universal solution to the tangent problem.

With this notation, we have a universal solution to the tangent problem. What do I mean by this? Recall that the tangent problem asks us to write the equation of the line tangent to the graph $y = f(x)$ at a point $(a, f(a))$. Since we're given a point on the tangent line for free (namely, $(a, f(a))$), and since we use $f'(a)$ to stand for the slope of the tangent line, the equation of the tangent line is

$$y - f(a) = f'(a)(x - a).$$

This is about all we talked about today in class in terms of new content. For the rest of the class period we just worked on homework problems. You can find these problems on the course webpage, as per usual.