PRACTICE WITH OPTIMIZATION

1. Announcements

We have our final fast approaching, and so I'm posting a practice final for you to work on. I'll also hold a review session next Sunday, March 19, sometime in the afternoon (more details on this later).

2. Discussion from Class

Today in class we mainly talked about homework problems. We came up with the following

Handy rule of thumb. More often than not, if you are asked to optimize area (with fixed perimeter) or volume (with fixed surface area), you'll be working on a closed interval. If you are asked to optimize perimeter (with fixed area) or surface area (with fixed volume), you're probably working on an open interval.

The one non-homework problem we did was an optimization problem for a continuous function on an open interval. Since all the homework problems were on a closed intervals, this was a good reminder of the procedure we follow when we find ourselves on an open interval

Example. Optimize $f(x) = x + \frac{1}{x}$ on the interval $(0, \infty)$.

Solution. Since $f(x)$ is continuous on the given interval (the only potential problem is $x = 0$, but this is outside our domain), the procedure is to find the critical points of $f(x)$ and then hope we can use the first derivative test for absolute extremes.

Now we know that $f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$, and so critical points are $x = \pm 1$ and $x = 0$. Of these, only $x = 1$ is in our given interval, and so we can use the first derivative test for absolute extremes. We choose a point to the left of $x = 1$ (say $x = \frac{1}{2}$) and a point to the right of $x = 1$ (say $x = 2$) to determine the sign of $f'(x)$ on the intervals $(0, 1)$ and $(1, \infty)$. Now we see

$$f'(\frac{1}{2}) = 1 - 4 < 0 \quad \text{and}$$

$$f'(2) = 1 - \frac{1}{4} > 0.$$

Hence the first derivative test for absolute extremes tells us that $x = 1$ is an absolute minimum. $\square$