HOMEWORK 1 SOLUTIONS

(21) Find the inverse of $f(x) = \sqrt{10-3x}$.

Solution. Since we are given $y = \sqrt{10-3x}$, solving for $f^{-1}(x)$ amounts to using our recipe: exchange the role of y and x in the equation, and solve for y. After exchanging y and x in the equation and squaring both sides we have $x^2 = 10 - 3y$, and solving for y gives

$$f^{-1}(x) = \frac{x^2 - 10}{-3}.$$

□

(22) Find the inverse of $f(x) = \dfrac{4x-1}{2x+3}$.

Solution. Since we are given $y = \dfrac{4x-1}{2x+3}$, to solve for $f^{-1}(x)$ we exchange y and x in the expression and solve for y. Exchanging y and x gives

$$x = \frac{4y-1}{2y+3},$$

and multiplying each side of this equality by $2y+3$ gives

$$2yx + 3x = (2y+3)x = 4y - 1.$$

Now moving all terms which have a y onto the left and all things without a y to the right gives

$$2yx - 4y = -1 - 3x,$$

and factoring a y on the left hand side (since each term has a y by our rearrangement!) gives

$$y(2x-4) = -1 - 3x.$$

Solving for y by dividing each side of this expression by $2x - 4$ gives

$$f^{-1}(x) = \frac{-1-3x}{2x-4}.$$

□

(23) Find the inverse of $f(x) = e^{x^3}$.

Solution. As before, we switch the role of y and x and solve for y. Hence we must find a solution for y when $x = e^{y^3}$.

$$x = e^{y^3} \Rightarrow \ln(x) = \ln(e^{y^3}) = y^3 \Rightarrow \sqrt[3]{\ln(x)} = y \Rightarrow f^{-1}(x) = \sqrt[3]{\ln(x)}.$$

□

(24) Find the inverse of $y = 2x^3 + 3$.

Solution. Following the same steps as always, we have to solve for y in $x = 2y^3 + 3$.

$$x = 2y^3 + 3 \Rightarrow x - 3 = 2y^3 \Rightarrow \frac{x-3}{2} = y^3 \Rightarrow \sqrt[3]{\frac{x-3}{2}} \Rightarrow f^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}.$$

□

(25) Find the inverse of $y = \ln(x+3)$.

Solution. Switching y and x gives the equation $x = \ln(y+3)$, which we use to solve for y:

$$x = \ln(y+3) \Rightarrow e^x = e^{\ln(y+3)} = y+3 \Rightarrow e^x - 3 = y \Rightarrow f^{-1}(x) = e^x - 3.$$

□

(26) Find the inverse of $\dfrac{1+e^x}{1-e^x}$.

Solution. We follow the same procedure as before, writing the given expression in the form $y = \cdots$, exchanging y and x, and solving for y. Hence we solve for y in

$$x = \frac{1+e^y}{1-e^y}.$$

Multiplying each side by $1-e^y$ gives $x(1-e^y) = x - xe^y$ on the left and $1+e^y$ on the right, and after moving all terms with a y to the right and all terms without a y to the left we are left with

$$x - 1 = e^y + xe^y = (1+x)e^y,$$

so that

$$e^y = \frac{x-1}{x+1}.$$

Taking the natural log of both sides gives

$$f^{-1}(x) = \ln\left(\frac{x-1}{x+1}\right).$$

□

(33) Find exact values for $\log_2(64)$ and $\log_6(\frac{1}{36})$.

Solution. Since $2^6 = 64$ we see $\log_2(64) = 6$, and since $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$ we have $\log_6(\frac{1}{36}) = -2$. □

(34) Find exact values for $\log_8(2)$ and $\ln(e^{\sqrt{2}})$.

Solution. Since $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$ we have $\log_8(2) = \frac{1}{3}$, and using properties of the natural logarithm we have $\ln(e^{\sqrt{2}}) = \sqrt{2}\ln(e) = \sqrt{2}$. □

(47) Solve for x in each of the following expressions:

(a) $2\ln(x) = 1$

Solution. Raising e to the left and right sides of this equation gives $(e^{\ln(x)})^2 = e^{2\ln(x)} = e^1$, which in turn gives $x^2 = e$. Since $x > 0$ (since we can't take log of a non-positive number), we have $x = \sqrt{e}$. □

(b) $e^{-x} = 5$

Solution. Taking natural logs of each side gives $-x = \ln(e^{-x}) = \ln(5)$, and so $x = -\ln(5)$. □

(48) Solve for x in each of the following expressions:

(a) $e^{2x+3} - 7 = 0$

Solution. After adding 7 to both sides of the equation we have $e^{2x+3} = 7$, and taking natural logs leaves $2x + 3 = \ln(7)$. Solving for x as usual gives

$$x = \frac{\ln(7) - 3}{2}.$$

□

(b) $\ln(5-2x) = -3$

Solution. Raising e to the left and right side of the equations gives $5-2x = e^{-3}$, and solving for x as per usual gives
$$x = \frac{e^{-3}-5}{-2}.$$
$\square$

(49) Solve for x in each of the following expressions:

(a) $2^{x-5} = 3$

Solution. Taking $\log_2$ of each side gives $x-5 = \log_2(2^{x-5}) = \log_2(3)$, and so $x = \log_2(3)+5$. $\square$

(b) $\ln(x) + \ln(x-1) = 1$

Solution. Using the properties of the natural log, this is equivalent to $\ln(x(x-1)) = 1$. Now raising e to each side of this equation gives $x(x-1) = e^{\ln(x(x-1))} = e^1$, and so using the quadratic formula we have two possible values of x:
$$x = \frac{1 \pm \sqrt{1+4e}}{2}.$$
However, our solution must be greater than 1 (so that we can take $\ln(x-1)$), and so we must have $x = \frac{1}{2}(1+\sqrt{1+4e})$. $\square$

(50) Solve for x in each of the following expressions:

(a) $\ln(\ln(x)) = 1$

Solution. We first raise e to the left and right hand sides of this expression, giving $\ln(x) = e^1 = e$. Repeating this once more gives
$$x = e^{e^1} = e^e.$$
$\square$

(b) $e^{ax} = Ce^{bx}$

Solution. Taking natural logs of both sides gives $ax = \ln(Ce^{bx}) = \ln(C) + \ln(e^{bx}) = \ln(C) + bx$, where here we've used the properties of natural log we talked about in class. Hence
$$x = \frac{\ln(C)}{a-b}.$$
$\square$

(E2) Suppose $k(x) = 2^{e^{3x}+1}$.

(a) Find functions $f(x), g(x)$ and $h(x)$ with $(f \circ g \circ h)(x) = k(x)$.

Solution. There are several different solutions to this problem, but one candidate is
$$f(x) = 2^x, g(x) = e^x + 1, h(x) = 3x.$$
$\square$

(b) Using f, g and h as above, compute $h(f(g(x)))$.

Solution. We see that $f(g(x)) = 2^{e^x+1}$, and so
$$h(f(g(x))) = 3\left(2^{e^x+1}\right).$$
$\square$

(c) Using f, g and h as before, find the inverse of $h(g(f(x)))$.

Solution. We see that

$$y = h(g(f(x))) = 3\left(e^{2^x} + 1\right),$$

so to find the inverse we do the normal trick: we exchange the role of y and x in the equation and solve for y. After some negotiating, this yields

$$(h(g(f(x))))^{-1} = \log_2(\ln\left(\frac{x}{3} - 1\right)).$$

One could also solve this problem by computing each of $h^{-1}(x), g^{-1}(x)$ and $f^{-1}(x)$, and then

$$(h(g(f(x))))^{-1} = (f^{-1} \circ g^{-1} \circ h^{-1})(x).$$

□

(E3) Consider the graph of the function given in Problem 6 of Section 1.3 (see page 46 in *Stewart*).

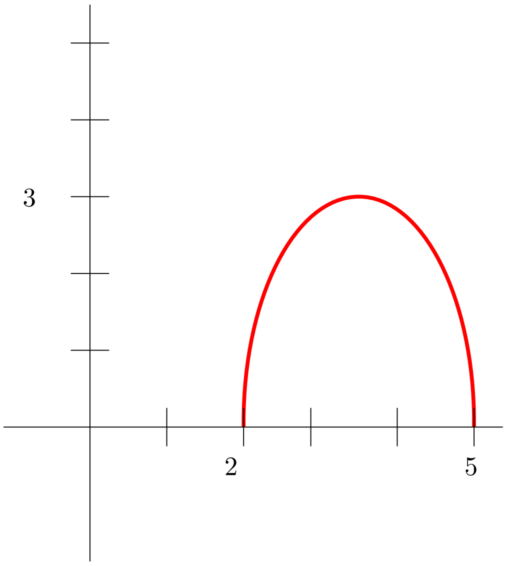

FIGURE 1. The graph of $f(x)$

(a) Compute $f^{-1}(3)$ and $f^{-1}(0)$.

Solution. From the graph, it seems $f^{-1}(3) = 3.5$ and $f^{-1}(0) = \{2, 5\}$ □

(b) Graph $f^{-1}(x)$.

Solution. The graph of $f^{-1}(x)$ (in blue) is shown on the same axes as the original graph (in red)

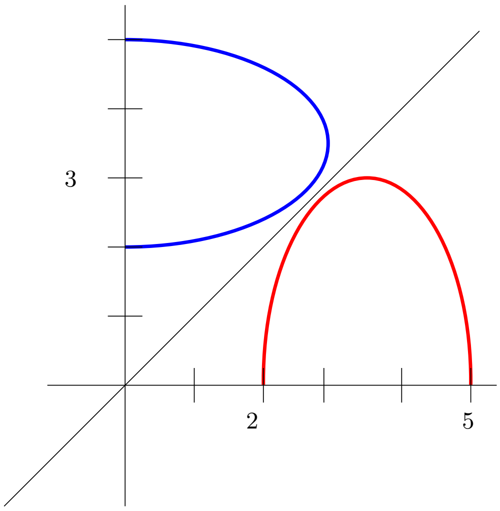

FIGURE 2. The graph of both $f(x)$ and $f^{-1}(x)$

□

(c) Is $f^{-1}(x)$ a function? Justify your claim.

Solution. We can see that the blue graph fails the vertical line test, and so $f^{-1}(x)$ is not a function.

□