HOMEWORK 5 SOLUTIONS

Instructions: Compute the following functions. You may not use a calculator except where explicitly stated. You are encouraged to use the power, product, quotient, and chain rule, together with the derivatives of exponential and trig functions we know, to solve these problems. My advice is to use the power rule (or exponential or trigonometric rules) for problems 1-25, the product or quotient rule for problems 26-36, and the chain rule for problems 37-48. Problems 49-54 I'll leave to your discretion.

(1) $\frac{d}{dx}\left[x+\sqrt{x}\right] = 1 + \frac{1}{2\sqrt{x}}$

(2) $\frac{d}{dx}\left[x^4\right] = 4x^3$

(3) $\frac{d}{dx}\left[\sin(x)+\cos(x)\right] = \cos(x) - \sin(x)$

(4) $\frac{d}{dx}\left[x^2 + \frac{1}{x}\right] = 2x - \frac{1}{x^2}$

(5) $\frac{d}{dx}\left[\sqrt[5]{x} + \frac{1}{\sqrt[3]{x}}\right] = \frac{1}{5}x^{-\frac{4}{5}} - \frac{1}{3}x^{-\frac{4}{3}}$

(6) $\frac{d}{dx}\left[2x^2+5x-1\right] = 4x+5$

(7) $\frac{d}{dx}\left[x^{1000}-1\right] = 1000x^{999}$

(8) $\frac{d}{dx}\left[x\sqrt{x}\right] = \frac{3}{2}x^{\frac{1}{2}} = \frac{3\sqrt{x}}{2}$

(9) $\frac{d}{dx}\left[\frac{x^2+1}{x}\right] = 1 - \frac{1}{x^2}$

(10) $\frac{d}{dx}\left[x^5+x^4+x^3+x^2+x+1\right] = 5x^4+4x^3+3x^2+2x+1$

(11) $\frac{d}{dx}\left[(x+1)(x^2+3)\right] = 3x^2+2x+3$

(12) $\frac{d}{dx}\left[x^7 - 3x^4 + 2x\right] = 7x^6 - 12x^3 + 2$

(13) $\frac{d}{dx}\left[\frac{1}{x} - \frac{1}{x^2}\right] = -\frac{1}{x^2} + 2\frac{1}{x^3}$

(14) $\frac{d}{dx}\left[x - \frac{1}{x}\right] = 1 + \frac{1}{x^2}$

(15) $\frac{d}{dx}\left[x^{100} - 5x^{20} + 100x\right] = 100x^{99} - 100x^{19} + 100$

(16) $\frac{d}{dx}\left[\frac{1}{x^5} + \sqrt{x} + 3\right] = -5\frac{1}{x^6} + \frac{1}{2\sqrt{x}}$

(17) $\frac{d}{dx}\left[e^x - x^2\right] = e^x - 2x$

(18) $\frac{d}{dx}\left[\sin(x) + \tan(x) + 5^x\right] = \cos(x) + \sec^2(x) + \ln(5)5^x$

(19) $\frac{d}{dx}\left[e^x + \sqrt{x} - \sec(x)\right] = e^x + \frac{1}{2}x^{-\frac{1}{2}} - \sec(x)\tan(x)$

(20) $\frac{d}{dx}\left[2^x\right] = \ln(2)2^x$

(21) $\frac{d}{dx}\left[3^x - e^x + \frac{1}{x}\right] = \ln(3)3^x - e^x - \frac{1}{x^2}$

(22) $\frac{d}{dx}\left[1^x + 2^x + 3^x + e^x\right] = \ln(1)1^x + \ln(2)2^x + \ln(3)3^x + e^x = \ln(2)2^x + \ln(3)3^x + e^x$

(23) $\frac{d}{dx}\left[-\cos(x)\right] = \sin(x)$

(24) $\frac{d}{dx}\left[\sec(x) + x^2 - \sqrt[3]{x^2}\right] = \sec(x)\tan(x) + 2x - \frac{2}{3}x^{-\frac{1}{3}}$

(25) $\frac{d}{dx}\left[(2x)^3\right] = \frac{d}{dx}\left[8x^3\right] = 24x^2$

(26) $\frac{d}{dx}\left[xe^x\right] = xe^x + e^x$

(27) $\frac{d}{dx}\left[x^2e^x + \frac{e^x}{x}\right] = 2xe^x + x^2e^x - \frac{xe^x - e^x}{x^2}$

(28) $\frac{d}{dx}\left[\frac{x}{x+1}\right] = \frac{(x+1)(1)-(x)(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$

(29) $\frac{d}{dx}\left[\frac{3+x}{1-3x}\right] = \frac{(1-3x)(1)-(3+x)(-3)}{(1-3x)^2} = \frac{10}{(1-3x)^2}$

(30) $\frac{d}{dx}\left[\frac{2x+5}{-x+2}\right] = \frac{(-x+2)(2)-(2x+5)(-1)}{(-x+2)^2} = \frac{9}{(2-x)^2}$

(31) $\frac{d}{dx}\left[\frac{e^x}{\sin(x)}\right] = \frac{\sin(x)e^x - e^x\cos(x)}{\sin^2(x)}$

(32) $\frac{d}{dx}\left[e^x\tan(x)\right] = e^x\tan(x) + e^x\sec^2(x)$

(33) $\frac{d}{dx}\left[(x+1)(e^x+3)\right] = (x+1)(e^x) + (e^x+3)(1)$

(34) $\frac{d}{dx}\left[\sqrt{x}(\sin(x)+x)\right] = \sqrt{x}(\cos(x)+1) + \frac{1}{2}x^{-\frac{1}{2}}(\sin(x)+x)$

(35) $\frac{d}{dx}\left[\frac{\sin(x)+\cos(x)}{\sin(x)-\cos(x)}\right] = \frac{(\sin(x)-\cos(x))(\cos(x)-\sin(x))-(\sin(x)+\cos(x))(\cos(x)+\sin(x))}{(\sin(x)-\cos(x))^2}$

(36) $\frac{d}{dx}\left[\frac{2^x}{x^3+x^2+x-\sqrt{x}}\right] = \frac{(x^3+x^2+x-\sqrt{x})(\ln(2)2^x) - 2^x(3x^2+2x+1-\frac{1}{2}x^{-\frac{1}{2}})}{(x^3+x^2+x-\sqrt{x})^2}$

(37) $\frac{d}{dx}\left[\sqrt{1+2x}\right] = \frac{1}{2\sqrt{1+2x}}\cdot 2 = \frac{1}{\sqrt{1+2x}}$

(38) $\frac{d}{dx}\left[\sin(\cos(x))\right] = \cos(\cos(x))(-\sin(x)) = -\sin(x)\cos(\cos(x))$

(39) $\frac{d}{dx}\left[e^{e^x}\right] = e^{e^x}e^x$

(40) $\frac{d}{dx}\left[2^{\sin(x)}\right] = \ln(2)2^{\sin(x)}(\cos(x)) = \ln(2)\cos(x)2^{\sin(x)}$

(41) $\frac{d}{dx}\left[\sin(3x)\right] = \cos(3x)(3) = 3\cos(3x)$

(42) $\frac{d}{dx}\left[e^{x^2}\right] = e^{x^2}(2x) = 2xe^{x^2}$

(43) $\frac{d}{dx}\left[\tan(e^x)\right] = \sec^2(e^x)\cdot e^x$

(44) $\frac{d}{dx}\left[\sqrt[3]{\sin(x)}\right] = \frac{1}{3}(\sin(x))^{-\frac{2}{3}}(\cos(x)) = \frac{\cos(x)}{3}(\sin(x))^{-\frac{2}{3}}$

(45) $\frac{d}{dx}\left[(x^2+1)^4\right] = 4(x^2+1)^3(2x) = 8x(x^2+1)^3$

(46) $\frac{d}{dx}\left[e^{3x^3+1}\right] = e^{3x^3+1}(9x^2) = 9x^2e^{3x^3+1}$

(47) $\frac{d}{dx}\left[e^{\sqrt{x^2+x}}\right] = e^{\sqrt{x^2+x}}(\frac{1}{2\sqrt{x^2+x}})(2x+1)$

(48) $\frac{d}{dx}\left[\sqrt{\sin(3x)}\right] = \left(\frac{1}{2\sqrt{\sin(3x)}}\right)(\cos(3x))(3)$

(49) $\frac{d}{dx}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}\right] = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}(1+\frac{1}{2\sqrt{x+\sqrt{x}}})(1+\frac{1}{2\sqrt{x}})$

(50) $\frac{d}{dx}\left[\frac{x^3+\sqrt{x}}{x}\right] = 2x - \frac{1}{2}x^{-\frac{3}{2}}$

(51) $\frac{d}{dx}\left[(x+1)(x^2+3)^4\right] = (x+1)(4(x^2+3)^3(2x)) + (x^2+3)^4$

(52) $\frac{d}{dx}\left[\sqrt{e^x}\right] = \frac{1}{2\sqrt{e^x}}e^x$

(53) $\frac{d}{dx}\left[\sin^2(x)+\cos^2(x)\right] = \frac{d}{dx}\left[1\right] = 0$

(54) $\frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right] = \frac{d}{dx}\left[\tan(x)\right] = \sec^2(x)$