HOMEWORK 6 SOLUTIONS

(3.6.3) Compute $\dfrac{dy}{dx}$ for the graph of $x^3 + x^2y + 4y^2 = 6$.

Solution. We use implicit differentiation, computing the derivative of the left- and right-hand sides and then solving for $\dfrac{dy}{dx}$. For simplicity I'll write y' instead of $\dfrac{dy}{dx}$.

Using the power rule, product rule, and chain rule, the left hand side has derivative $3x^2 + 2xy + x^2y' + 8yy'$. The right hand side has derivative 0. This means we have

$$3x^2 + 2xy + x^2y' + 8yy' = 0.$$

Moving terms without a y' to the right gives

$$y'(x^2 + 8y) = -3x^2 - 2xy,$$

so that

$$y' = \frac{-3x^2 - 2xy}{x^2 + 8y}.$$

□

(3.6.5) Compute $\dfrac{dy}{dx}$ for the graph of $x^2y + xy^2 = 3x$.

Solution. Again, we use implicit differentiation and the abbreviation $y' = \dfrac{dy}{dx}$.

The power rule, product rule, and chain rules tell me the left hand side has derivatives $2xy + x^2y' + y^2 + 2xyy'$. The power rule says the right hand side has derivative 3. Setting these equal gives

$$2xy + x^2y' + y^2 + 2xyy' = 3.$$

Moving all terms with a y' to the left and all terms without a y' to the right gives the equality

$$y'(x^2 + 2xy) = 3 - 2xy - y^2,$$

which in turn gives

$$y' = \frac{3 - 2xy - y^2}{x^2 + 2xy}.$$

□

(3.6.6) Compute $\dfrac{dy}{dx}$ for the graph of $y^5 + x^2y^3 = 1 + ye^{x^2}$.

Solution. The left hand side of the expression has derivative $5y^4y' + 2xy^3 + 3x^2y^2y'$, and the right hand side has derivative $2x\ ye^{x^2} + y'e^{x^2}$ (notice: this required the chain rule and the product rule). Setting these equal gives

$$5y^4y' + 2xy^3 + 3x^2y^2y' = 2x\ ye^{x^2} + y'e^{x^2},$$

which can be rearranged to give

$$y'(5y^4 + 3x^2y^2 - e^{x^2}) = 2xye^{x^2} - 2xy^3.$$

This means

$$y' = \frac{2xye^{x^2} - 2xy^3}{5y^4 + 3x^2y^2 - e^{x^2}}.$$

□

(3.6.8) Compute $\dfrac{dy}{dx}$ for the graph of $1 + x = \sin(xy^2)$.

Solution. Clearly the left hand side has derivative 1, while the chain rule tells us the derivative of the right hand side is $\cos(xy^2)\left(y^2 + 2xyy'\right)$. This gives the equality

$$1 = y^2 \cos(xy^2) + 2xyy' \cos(xy^2)$$

which can be rearranged to give

$$1 - y^2 \cos(xy^2) = 2xyy' \cos(xy^2).$$

This means

$$y' = \frac{1 - y^2 \cos(xy^2)}{2xy \cos(xy^2)}.$$

□

(3.6.21(a)) Find the equation of the line tangent to the graph of $y^2 = 5x^4 - x^2$ at the point $(1, 2)$.

Solution. To find the equation of the tangent line we must compute the slope of the tangent at $(1, 2)$. This means we need to solve for y' and evaluate it at $(1, 2)$. Once we know the slope of the tangent line, we can use point-slope form to write the equation of the tangent.

To solve for y' we use implicit differentiation. You can verify that

$$y' = \frac{20x^3 - 2x}{2y},$$

and hence the slope of the tangent at $(1, 2)$ is $\frac{20-2}{4} = \frac{18}{4} = \frac{9}{2}$. Hence the equation of the tangent is

$$y - 2 = \frac{9}{2}(x - 1).$$

□

(3.6.22(a)) Find the equation of the line tangent to the graph of $y^2 = x^3 + 3x^2$ at the point $(1, -2)$.

Solution. Just like in the last problem, to find the equation of the tangent line we must compute the slope of the tangent at $(1, -2)$. This means we need to solve for y' and evaluate it at $(1, -2)$. Once we know the slope of the tangent line, we can use point-slope form to write the equation of the tangent.

To solve for y' we use implicit differentiation. You can verify that

$$y' = \frac{3x^2 + 6x}{2y},$$

and hence the slope of the tangent at $(1, -2)$ is $\frac{3+6}{-4} = -\frac{9}{4}$. Hence the equation of the tangent is

$$y + 2 = -\frac{9}{4}(x - 1).$$

□