FAKE FINAL

- Complete the following fake problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes fake exam. No calculators or other electronic aids will be permitted.
- In order to receive full fake credit, please show all of your work and justify your fake answers. You do not need to simplify your fake answers unless specifically fake instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this fake exam. Do not unstaple or detach pages from this fake exam.
- Please sign the following:

  "On my honor, I have neither given nor received any aid on this fake examination. I have furthermore abided by all other aspects of the honor code with respect to this fake examination."

Signature: ________________________________

The following boxes are strictly for fake grading purposes. Please do not mark.

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(1) Complete each of the following sentences.
  (a) The extreme value theorem states

  (b) Critical points of a function $f(x)$ are defined as points where

  (c) The equation of the line tangent to the graph of $f(x)$ at a point $(a, f(a))$ is

(2) Determine whether each statement is true or false; unless otherwise stated, any function below is arbitrary. **If the statement is true, cite your reasoning. If it is false, provide a counterexample.**

(a) If $f(a) < 0$ and $f(b) > 0$, then there exists some value c between a and b with $f(c) = 0$.

(b) A continuous function on an open interval must have an absolute maximum.

(c) $f'(x)$ has the same domain as $f(x)$.

(3) For each of the following conditions, provide an example of a function $f(x)$ which satisfies the given condition. Unless otherwise indicated, you may express your function either with an explicit formula or by a graph.

(a) $f(x)$ has neither an absolute minima nor an absolute maxima on its domain

(b) $f(x)$ has the property that $f'(x) = \dfrac{1}{1+x^2}$

(c) $\lim_{h\to 0} f(h) = f(0)$ yet $\lim_{h\to 0} \dfrac{f(0+h)-f(0)}{h}$ does not exist.

(4) Show there exists a solution to the equation $\ln(x) = \sin\left(\frac{\pi}{2}x\right)$ in the interval $(0, \infty)$.

(5) Find 2 positive numbers whose product is 3 and such that the sum of their squares is minimum.

(6) Find the triangle with minimum area which is enclosed by the x-axis, the y-axis, and a line passing through the point $(1, 2)$.

(7) Find the absolute maximum and minimum of the function $f(x) = xe^{-x}$ on the interval $[0, 2]$. You may use the fact that $e > 2$ in your analysis.

(8) Use the definition of the derivative as a limit to compute

(a) $\dfrac{d}{dx}\left[x^{-\frac{1}{2}}\right]$

(b) $\dfrac{d}{dx}\left[e^{x}\right]$ (here you are encouraged to use the fact that $\lim_{h\to 0}\dfrac{e^{h}-1}{h}=1$).

(9) Compute the following derivatives. In this problem, you may use any rules we discussed in class.

(a) $\dfrac{d}{dx}\left[2^{3^x}\right]$

(b) $\dfrac{d}{dx}\left[\cos(\arcsin(x))\right]$

(c) $\dfrac{d}{dx}\left[\ln\left(\sec^2(x)+1\right)\right]$

(d) $\dfrac{d}{dx}\left[\sqrt{e^{x^3-3x}}\right]$

(10) (a) Find the equation of the tangent line to the graph of y at the point (x_0, y_0), where y and x satisfy the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(b) When $a = 2$ and $b = 4$, use linearization to approximate a y-value corresponding to $x = 2.1$.