# QUIZ 1 SOLUTIONS

*Instructions:* Complete the following problems. Be sure to show your work when you make 'significant' leaps in problem solving. Answers which are not accompanied by justification will receive little or no credit.

(1) (20 points)

- Find the inverse of the function $f(x) = \dfrac{x+1}{x}$.

*Solution.* We have $y = \dfrac{x+1}{x}$, and the technique for finding $f^{-1}(x)$ is to exchange $y$ and $x$ in this expression and solve for $y$. Exchanging $y$ and $x$ gives $x = \dfrac{y+1}{y}$, and now multiplying through by $y$ gives $xy = y + 1$. Moving all terms with a $y$ onto the left and all terms without a $y$ to the right gives $xy - y = 1$, and factoring out $y$ on the left yields $y(x-1) = 1$. Solving for $y$ gives

$$f^{-1}(x) = \frac{1}{x-1}.$$

$\square$

- Find the inverse of the function $f(x) = \ln(x^3)$.

*Solution.* As before, we take the given expression $y = \cdots$, exhange $y$ and $x$, and solve for $y$. Hence we have $x = \ln(y^3)$, and raising $e$ to both sides gives the equality $e^x = y^3$. Taking cube roots gives

$$f^{-1}(x) = \sqrt[3]{e^x}.$$

$\square$

(2) (10 points) Compute the following quantities

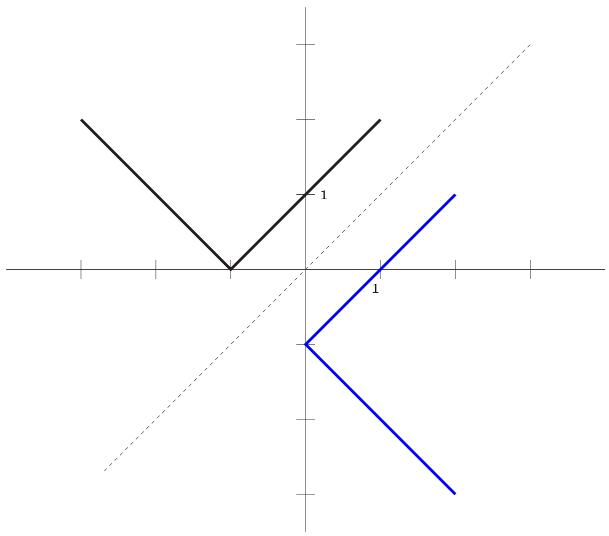
- $\ln(1)$

*Solution.* We know from class that $\ln(1) = 0$. □

- $\log_3(9)$

*Solution.* Since $3^2 = 9$, we have $\log_3(9) = 2$. □

(3) (30 points) Consider the graph of the function $f(x)$ given below.




- What is $f^{-1}(0)$?

*Solution.* Since $f^{-1}(0)$ is the set of all those inputs which have output 0, we see that $f^{-1}(0) = -1$. □

- On the graph above, sketch $f^{-1}(x)$.

*Solution.* See the graph above (the inverse is in blue). □

- Is $f^{-1}$ a function? Why or why not?

*Solution.* We see above that the graph of the inverse fails the vertical line test, and so $f^{-1}(x)$ is not a function. □

(4) (20 points)

- Find functions $f(x)$ and $g(x)$ with $\sqrt{x^3+1} = (f \circ g)(x)$.

*Solution.* By inspection we see that

$$f(x) = \sqrt{x}$$

and

$$g(x) = x^3 + 1.$$

$\square$

- Find functions $f(x), g(x)$ and $h(x)$ with $\tan(\cos(\sin(x))) = (f \circ g \circ h)(x)$.

*Solution.* Again by inspection we have

$$f(x) = \tan(x)$$

$$g(x) = \cos(x)$$

and

$$h(x) = \sin(x).$$

$\square$

(5) (20 points)

- Find the slope of the line passing through $(2, 3)$ and $(4, 11)$.

*Solution.* Slope is given by the change in $y$-coordinates divided by the change in $x$-coordinates (the infamous 'rise-over-run'):

$$m = \frac{11-3}{4-2} = \frac{8}{2} = 4.$$

$\square$

- Find the equation of the line passing through $(2, 3)$ and $(4, 11)$.

*Solution.* Since we know the slope of the line and a point on the line, we can write the equation using point-slope form. Three correct (and equivalent) answers are

$$y - 3 = 4(x-2)$$

$$y - 11 = 4(x-4)$$

$$y = 4x - 5$$

$\square$