QUIZ 2

Instructions: Complete the following problems. Be sure to show your work when you make 'significant' leaps in problem solving. Answers which are not accompanied by justification will receive little or no credit.

(1) (30 pts) Evaluate $\lim_{x\to 2} \dfrac{3x^2-4x-4}{x^2+3x-10}$, if it exists. If it does not exist, explain why.

Solution. We notice that the numerator can be factored as $(3x+2)(x-2)$ and the denominator can be factored as $(x+5)(x-2)$, and so for $x \neq 2$ we have

$$\frac{3x^2-4x-4}{x^2+3x-10} = \frac{(3x+2)(x-2)}{(x+5)(x-2)} = \frac{3x+2}{x+5}.$$

Since these functions agree away from $x=2$, we can evaluate the limit as x approaches 2 of the left-hand side by evaluating the limit as x approaches 2 of the right-hand side. Since the right hand side is continuous at $x=2$, we can evaluate this latter limit by evaluating at 2. Therefore, we have

$$\lim_{x\to 2} \frac{3x^2-4x-4}{x^2+3x-10} = \lim_{x\to 2} \frac{3x+2}{x+5} = \frac{8}{7}.$$

□

(2) (30 pts) Evaluate $\lim_{h\to 1} \dfrac{\sqrt{10-h}-3}{h-1}$, if it exists. If it does not exist, explain why.

Solution. We know from class that the way to evaluate this limit is by 'rationalizing the numerator' by multiplying by the conjugate:

$$\frac{\sqrt{10-h}-3}{h-1} \cdot \frac{\sqrt{10-h}+3}{\sqrt{10-h}+3} = \frac{10-h-9}{(h-1)(\sqrt{10-h}+3)}$$

for values of $h \neq 1$. Now we get some nice cancellation which will let us evaluate the limit easily:

$$\begin{aligned}\lim_{h\to 1} \frac{\sqrt{10-h}-3}{h-1} &= \lim_{h\to 1} \frac{10-h-9}{(h-1)(\sqrt{10-h}+3)} \\ &= \lim_{h\to 1} \frac{-h+1}{(h-1)(\sqrt{10-h}+3)} = \lim_{h\to 1} \frac{-1}{\sqrt{10-h}+3} \\ &= \frac{-1}{\sqrt{10-1}+3} = -\frac{1}{6}.\end{aligned}$$

□

(3) (30 pts) Evaluate $\lim_{x\to 1} \dfrac{e^x}{\ln(x)}$, if it exists. If it does not exist, explain why.

Solution. Since e^x is a continuous function everywhere, the limit of the numerator is just the value of e^x at 1:

$$\lim_{x\to 1} e^x = e^1 = e.$$

In the denominator, since $\ln(x)$ is continuous on its domain, it is also continuous at 1. Again, this means the limit at 1 is just the value at 1:

$$\lim_{x\to 1} \ln(x) = \ln(1) = 0.$$

In class we said that if we evaluate the limit of the numerator is a non-zero number and the limit of the denominator is zero, then the limit of the quotient function does not exist:

$$\lim_{x\to 1} \frac{e^x}{\ln(x)} \text{ does not exist.}$$

□

(4) (10 pts) In class we said that $\lim_{x\to 0} \left[\sin\left(\dfrac{1}{x}\right)\right]$ does not exist. Suppose we define a function $f(x)$ by the rule

$$f(x) = \begin{cases} \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Is $f(x)$ continuous at 0? Justify your answer. (An answer with little or incorrect justification will receive severely reduced credit)

Solution. Since $f(x) = \sin(\frac{1}{x})$ for $x \neq 0$, we know $\lim_{x\to 0} f(x) = \lim_{x\to 0} \sin(\frac{1}{x})$. As $\sin(\frac{1}{x})$ does not have a limit at 0, we have $\lim_{x\to 0} f(x)$ does not exist. In class we said that if a function does not have a limit at a point a, then it cannot be continuous at a. Since $f(x)$ does not have a limit at 0, it follows, then, that $f(x)$ cannot be continuous at 0. □