QUIZ 3

Instructions: Complete the following problems. Be sure to show your work when you make 'significant' leaps in problem solving. Answers which are not accompanied by justification will receive little or no credit.

(1) (20 pts) For a function $f(x)$, the derivative $f'(x)$ is defined to be a certain limit. Write that limit.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

(2) (25 pts) For $f(x) = (1+x)^2$, compute $f'(x)$ by evaluating an appropriate limit.

We compute the derivative using the definition.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(1+x+h)^2 - (1+x)^2}{h} \\
&= \lim_{h\to 0} \frac{1 + x + h + x + x^2 + hx + h + hx + h^2 - (1 + 2x + x^2)}{h} \\
&= \lim_{h\to 0} \frac{1 + 2x + 2h + 2hx + x^2 + h^2 - 1 - 2x - x^2}{h} \\
&= \lim_{h\to 0} \frac{2h + 2hx + h^2}{h} = \lim_{h\to 0} \frac{h(2 + 2x + h)}{h} = \lim_{h\to 0} [2 + 2x + h] \\
&= 2 + 2x
\end{aligned}$$

(3) (25 pts) For $f(x) = \frac{x}{x+1}$, compute $f'(x)$ by evaluating an appropriate limit.

The derivative is

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h} \\
&= \lim_{h\to 0} \frac{\frac{x+1}{x+1}\cdot\frac{x+h}{x+h+1} - \frac{x}{x+1}\cdot\frac{x+h+1}{x+h+1}}{h} = \lim_{h\to 0} \frac{(x+1)(x+h)-x(x+h+1)}{h(x+1)(x+h+1)} \\
&= \lim_{h\to 0} \frac{x^2+xh+x+h-x^2-xh-x}{h(x+1)(x+h+1)} = \lim_{h\to 0} \frac{h}{h(x+1)(x+h+1)} \\
&= \lim_{h\to 0} \frac{1}{(x+1)(x+h+1)} = \frac{1}{(x+1)^2}
\end{aligned}
$$

(4) (30 pts) The graph of $g(x)$ is

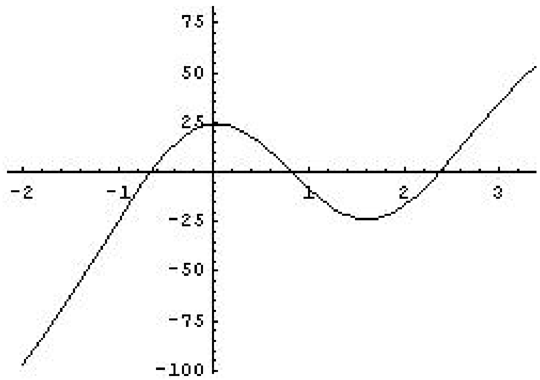
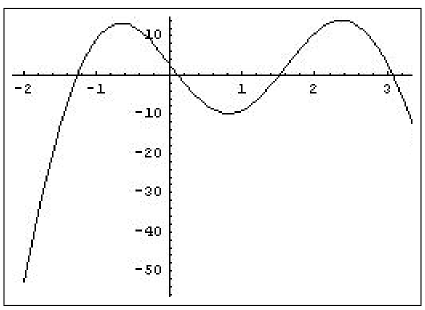
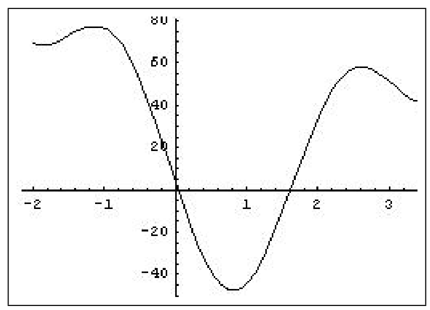
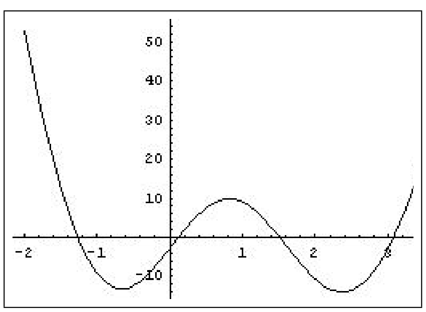


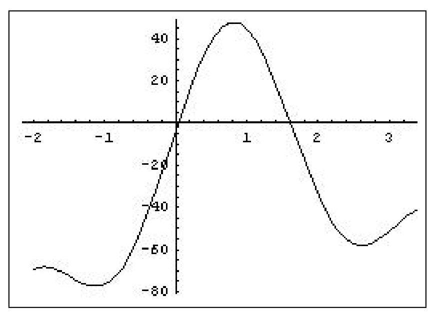
FIGURE 1. The graph of $g(x)$

Which of the following 4 graphs could be the graph of $\frac{d}{dx}[g(x)]$? Why? Remember, an answer with no justification will receive severely reduced credit, if any at all.

(A)


(B)


(C)


(D)


The graph of $g(x)$ has a horizontal tangent at approximately $x \approx 0$ and $x \approx 1.6$, so the graph of $g'(x)$ should pass through $y = 0$ at these points. This means the derivative is either (B) or (D). But since slopes of tangents to the graph of $g(x)$ on the left of $x = 0$ are positive, the graph of $g'(x)$ needs to be positive to the left of $x = 0$. Hence (B) must be the graph of the derivative.

If you finish early, you can use this space to draw a picture.

Figure 2. Bred for its skills in magic