QUIZ 6 SOLUTIONS

Instructions: Complete the following problems. You may use the derivative shortcuts developed in class to answer these problems, though you are encouraged to show as much work as possible so that partial credit may be awarded.

(1) (30 pts) Find the absolute maximum of the function $f(x) = 2\ln(x) - x^2$ on $(0, \infty)$. Be sure to justify why your answer is a maximum.

We know that

$$f'(x) = \frac{2}{x} - 2x = \frac{2 - 2x^2}{x} = \frac{2(1-x)(1+x)}{x},$$

so the critical points are $x = 0$ and $x = \pm 1$. Only $x = 1$ is in our interval (happily). Now we sample the sign of $f'(x)$ at a point to the left and a point to the right of $x = 1$. At $\frac{1}{2}$ we have $f'(\frac{1}{2}) = \frac{2-2\ \frac{1}{4}}{\frac{1}{2}} > 0$, and at 2 we have $f'(2) = \frac{2-8}{2} < 0$. Hence the first derivative test for absolute extrema tells us $x = 1$ is an absolute maximum of the function.

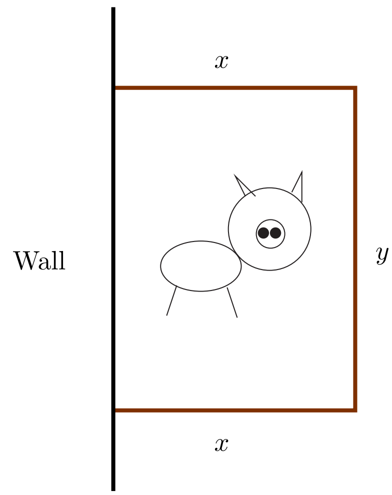
(2) (30 pts) Find the absolute extrema (maxima and minima) of the function $f(x) = 2x^3 + 3x^2 - 12x + 1$ on the interval $[0, 2]$.

Since our function is continuous on the closed interval, we proceed as per usual.
We see $f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x + 2)(x - 1)$, so our critical points are $x = -2$ and $x = 1$. Since -2 is outside the interval, however, $x = 1$ is the only one that counts.
We now compare function values at critical points and endpoints of the interval.

x	$f(x)$
0	1
1	$2 + 3 - 12 + 1 = -6$
2	$2(2)^3 + 3(2)^2 - 12(2) + 1 = 16 + 12 - 24 + 1 = 5$

Hence our function has an absolute maximum at $x = 2$ and am absolute minimum at $x = 1$.

(3) (40 pts) Old MacDonald had a farm, and on that farm he had an old stone wall running along the western edge of his fields. He also has 1200 feet of fencing that he'd like to use to make a rectangular pen for his pigs. Cleverly, he decides that he'll use that old stone wall for one side of the pen, so that he'll use his fencing for the other three sides. What is the maximum area that Old MacDonald's pigpen can have?



My picture illustrates the situation. I want to maximize $A = xy$, subject to the restraint $2x + y = 1200$. Hence $y = 1200 - 2x$, and so $A = x(1200 - 2x) = 1200x - 2x^2$. I'd like to optimize this function on the interval $[0, 600]$.
Now $A'(x) = 1200 - 4x$, so that the only critical point is 300. We now compare function values at critical points and endpoints of the interval.

x	$A(x)$
0	1
300	$300(600) = 180000$
600	0

Hence our function has an absolute maximum at $x = 300$, and the maximum are is $180,000$.