MIDTERM 1

- Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- Please sign the following:

  "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Name: ____________________________________

Signature: ____________________________________

The following boxes are strictly for grading purposes. Please do not mark.

1	15 pts	
2	20 pts	
3	25 pts	
4	30 pts	
5	35 pts	
6	30 pts	
Total	155 pts	

(1) (15pts) Complete each of the following sentences.

- Geometrically, the derivative $f'(a)$ is

- In class we defined a function $f(x)$ to be continuous at a point a if

- Given a function $f(x)$ and a point $(a, f(a))$ on the graph of $f(x)$, the tangent problem asks

(2) (20 pts) Determine whether each statement is true or false for arbitrary functions $f(x)$ and $g(x)$. **If the statement is true, cite your reasoning. If it is false, provide a counterexample.**

- If $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, then $\lim_{x\to 0}[f(x)g(x)]$ does not exist.

- If $f(0)$ is undefined, then $\lim_{x\to 0} f(x)$ does not exist.

(3) (25 pts)

- Draw the graph of a function $f(x)$ that satisfies $f'(0) = f'(10) = 0$, $f(0) = 5$, and $f(10) = 0$.

- On the same set of axes as above, graph $f^{-1}(x)$. Be sure to label which graph is $f(x)$ and which is $f^{-1}(x)$.

- Is $f^{-1}(x)$ a function? Explain why or why not.

(4) (30 pts) Suppose $f(x)$ is a function that is continuous everywhere and satisfies $f(1) = -1$ and $f(5) = 10$.

- Is there a solution to the equation $f(x) = \log_5(x)$? If so, explain why. If not, give a counterexample.

- Compute $\lim_{x\to 1}\left[f(x) + 2^{f(x)}\right]$.

(5) (35 pts) Let $f(x) = \dfrac{5x+2}{2x+1}$.

- Compute $f'(0)$ by evaluating an appropriate limit.

- Write the equation of the line tangent to $y = f(x)$ at $(0, f(0))$.

- Find $f^{-1}(x)$.

(6) (30 pts)

- Evaluate $\lim_{x\to 2}\left[\frac{\left(\frac{x}{\sqrt{x+2}}-1\right)}{x-2}\right]$.

- For $f(x)=(1+x)^2$, compute $f'(1)$ by evaluating an appropriate limit.