MIDTERM 2

- Complete the following problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so.
- If you need extra room, use the back sides of each page or the scratch paper provided at the end of the exam. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- Please sign the following:

  "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Name: ____________________________________

Signature: ____________________________________

The following boxes are strictly for grading purposes. Please do not mark.

1	15 pts	
2	15 pts	
3	20 pts	
4	35 pts	
5	30 pts	
6	20 pts	
7	20 pts	
8	45 pts	
Total	200 pts	

(1) (15 pts) Complete each of the following sentences.

(a) The derivative of a function $f(x)$ is defined to be the limit

(b) The critical points of a function $f(x)$ are defined to be

(c) According to the calculus/geometry dictionary, the inequality $f''(x) > 0$ on (a, b) is equivalent to 2 things. They are

(2) (15 pts) Determine whether each statement is true or false for arbitrary functions $f(x)$ and $g(x)$. **If the statement is true, cite your reasoning. If it is false, provide a counterexample.**

(a) If $f(x)$ satisfies $f'(0) = 0$, then $f(x)$ has a local maximum or local minimum at $x = 0$.

(b) If a function is continuous at 0, then it is differentiable at 0.

(c) If $f'(a) = 0$, then $f''(a) = 0$ also.

(3) (20 pts)

(a) Give an example of a function $f(x)$ which satisfies $f(x) = -f''(x)$.

(b) In class we said a function can fail to be differentiable at a point in three different ways. Draw the graph of a function which exhibits all of these failures, labeling each of the offending points by its failure.

(4) (35 pts)

(a) Use the definition of the derivative as a limit to compute $\frac{d}{dx}[x^2]$.

(b) Use the definition of the derivative as a limit to compute $\frac{d}{dx}[x]$.

(c) Use your results from the previous two calculations, together with the chain rule, to prove that $\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}}$. You may not use the power rule. (Hint: Consider the equation $\left(\sqrt{x}\right)^2 = x$.)

(5) (30 pts) Compute the following. You do not have to use the definition of the derivative as a limit.

(a) $\dfrac{d}{dx}\left[\dfrac{x^3+x^2+\ln(x)}{e^{\cos(x)}}\right]$

(b) $\dfrac{d}{dx}\left[\arctan\left(\sqrt{1+\arcsin(x)}\right)\right]$

(6) (20 pts) For a particular function $f(x)$, the graph of $f'(x)$ is:

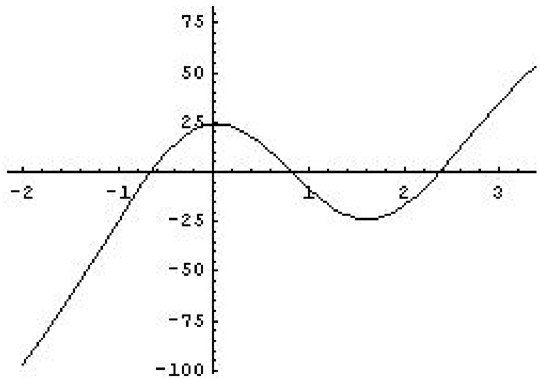
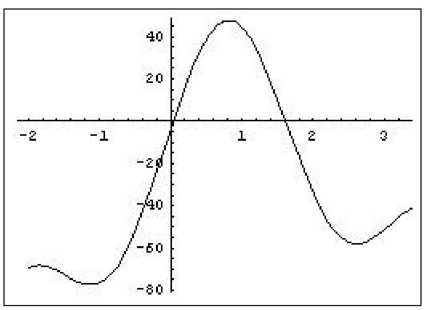
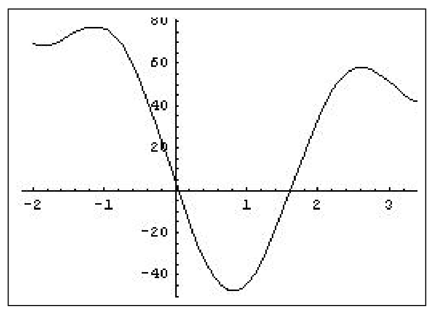
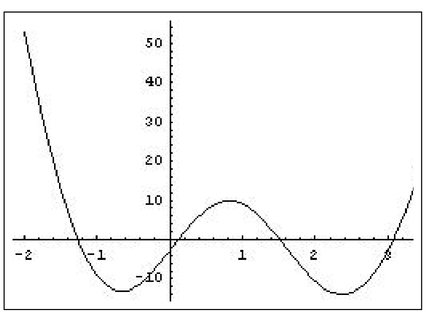


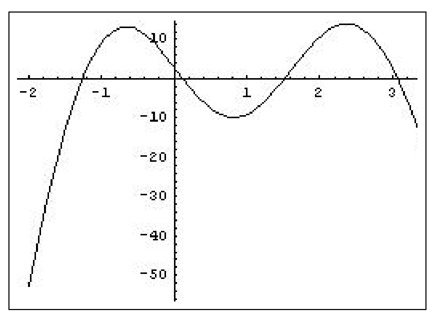
FIGURE 1. The graph of $f'(x)$

Which of the following 4 graphs could be the graph of $f(x)$? Why? An answer without sufficient justification will receive little or no credit.

(A)


(B)


(C)


(D)


(7) (20 pts) Find the equation of the line tangent to $x^3 - y^3 = -6xy$ at $(3, -3)$.

(8) (45 pts) Let $f(x) = 4x^5 + 5x^4 - 40x^3 + 17$.

(a) Compute $f'(x)$ and $f''(x)$. You do not have to use the definition of the derivative as a limit.

(b) List all possible extreme values of the function.

(c) Find intervals on which $f(x)$ is increasing and intervals on which $f(x)$ is decreasing.

(d) From your list of possible extreme values, determine which are actually local maxima and which are actually locally minima (for each, be sure to justify why it is a max/min).

(e) I was originally planning on asking you to find intervals on which $f(x)$ is concave up and intervals on which $f(x)$ is concave down. If I had asked that question, how would you have solved it?