FAKE MIDTERM 2

- Complete the following fake problems. You may use any result from class you like, but if you cite a theorem be sure to verify the hypotheses are satisfied.
- This is a closed-book, closed-notes fake exam. No calculators or other electronic aids will be permitted.
- In order to receive full fake credit, please show all of your work and justify your fake answers. You do not need to simplify your fake answers unless specifically fake instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this fake exam. Do not unstaple or detach pages from this fake exam.
- Please sign the following:

  "On my honor, I have neither given nor received any aid on this fake examination. I have furthermore abided by all other aspects of the honor code with respect to this fake examination."

  Signature: _____

The following boxes are strictly for fake grading purposes. Please do not mark.

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(1) Complete each of the following sentences.

(a) A function $f(x)$ is said to be strictly increasing if

(b) An inflection point is

(c) According to the chain rule, if $f(x)$, $g(x)$, and $h(x)$ are differentiable, then

$$\frac{d}{dx}\left[f(g(h(x)))\right] =$$

(2) Determine whether each statement is true or false for arbitrary functions $f(x)$ and $g(x)$. **If the statement is true, cite your reasoning. If it is false, provide a counterexample.**

(a) If $f'(x) > 0$ on the interval $(0, 2)$, then $f(x)$ is concave up on $(0, 2)$.

(b) If a function is differentiable at a point a, then it is continuous at a.

(3) For each of the following problems, provide an example of a function $f(x)$ which satisfies the following properties. You can either draw your function or give an explicit algebraic expression for your function.

(a) $f'(0) = f'(2) = 0$, yet f has neither a local maximum nor a local minimum at either 0 or 2.

(b) $f'(0) > 0$, $f''(0) < 0$, and $f(0) = 0$.

(4) Use the chain rule and the product rule (but not the quotient rule!) to show that

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}.$$

(5) Your friend has two functions, $f(x)$ and $g(x)$. He calls their product $h(x)$; i.e., $h(x) = f(x)g(x)$. One day he asks you to compute $g'(5)$, but only tells you

- $g(5) = 1$,
- $f(5) = 2$,
- $f'(5) = 3$,
- $h'(5) = 4$.

What is $g'(5)$? Be sure to explain your answer.

(6) Compute the following derivatives.

(a) $\dfrac{d}{dx}\left[\arctan\left(\sqrt{e^{\sin(x)}}\right)\right]$

(b) $\dfrac{d}{dx}\left[\ln(x^2+1)\sin(3x)\right]$

(7) Find the equation of the line tangent to $y^2 + xy + x^2 = e^{y-2} + 6$ at the point $(1, 2)$

(8) Suppose $f(x) = e^{x^3-1}$. Determine
- all critical points of $f(x)$,
- intervals on which $f(x)$ is increasing
- intervals on which $f(x)$ is decreasing
- intervals on which $f(x)$ is concave up
- intervals on which $f(x)$ is concave down