Number Systems

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Following are some of the different number systems discussed in the history of mathematics.

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The Number Sense

The number sense is not the ability to count, but the ability to recognize that something has changes in a small collection. Some animal species are capable of this.

The number of young that the mother animal has, if changed, will be noticed by all mammals and most birds. Mammals have more developed brains and raise fewer young than other species, but take better care of their young for a much longer period of time.

Many birds have a good number sense. If a nest contains four eggs, one can safely be taken, but when two are removed the bird generally deserts. The bird can distinguish two from three.[1]

An experiment done with a goldfinch showed the ability to distinguish piles of seed: three from one, three from two, four from two, four from three, and six from three. The goldfinch almost always confused five and four, seven and five, eight and six, and ten and six.

Another experiment involved a squire who was trying to shoot a crow which made its nest in the watchtower of his estate. The squire tried to surprise the crow, but at his approach, the crow would leave, watch from a distance, and not come back until the man left the tower. The squire then took another man with him to the tower. One man left and the other stayed to get the crow when it returned to the nest, but the crow was not deceived. The crow stayed away until the other man came out. The experiment was repeated the next day with three men, but the crow would not return to the nest. The following day, four men tried, but it was not until that next day with five men that the crow returned to the nest with one man still in the tower.[2]

In the insect world, the solitary wasp seemed to have the best number sense. �The mother wasp lays her eggs in individual cells and provides each egg with a number of live caterpillars on which the young feed when hatched. Some species of wasp always provide five, others twelve, and others as high as twenty-four caterpillars per cell. The solitary wasp in the genus Eumenus, will put five caterpillars in the cell if it is going to be a male (the male is smaller) and ten caterpillars in a female�s cell. This ability seems to be instinctive and not learned since the wasp�s behavior is connected with a basic life function.�[3]

One might think people would have a very good number sense, but as it turns out, people do not. �Experiments have shown that the average person has a number sense that is around four.�[4]

People groups in the world today that have not developed finger counting have a hard time discerning the quantity four. They tend to use the quantities one, two and many-which would include four.

�Small children around fourteen months of age will almost always notice something that is missing from a group that he or she is familiar with. The same age child can usually reassemble objects that have been separated into one group again. But the child�s ability to perceive numerical differences in the people or objects around him or her are very limited when the number goes beyond three or four.�[5]

So what separates people from the rest of the animal kingdom? It may include many things, but the ability to count is very much one of them. Counting, which usually begins at the end of our own hands or fingers, is usually taught by another person or possibly by circumstance. It is something that we should never take lightly for it has helped advance the human race in countless ways.

The number sense is something many creatures in this world have as well as well as we do.

Although, as we can see, our human ability is not much better than the common crow�s ability. We are born with the number sense, but we get to learn how to count.

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[1] Dantzig, p. 1.
[2] Dantzig, p. 3.
[3] Infrah, p. 4.
[4] Dantzig, p. 5.
[5] Infrah, p. 6.

Contributed by Bruce White

References:

1. Dantzig, Tobias. Number: The Language of Science. New York: Macmillan Company, 1930.
2. Ifrah, Georges. From One to Zero: A Universal History of Numbers. New York: Viking Penguin, Inc., 1985.

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Quipu - An Inca Counting System

Imagine, if you will, a highly advanced civilization. This civilization rules over a million or more people, they built vast cities, developed extensive road systems, treated their citizens fairly and constructed stone walls so tight not even a knife blade can pass between the huge boulders. Now imagine being able to do all this without a written language.

This was the ancient South American civilization of the Inca Empire. A highly developed civilization able to track all important facts required to rule such a vast empire. They did this using a memory tool made of knotted strings called a quipu. The men in charge of maintaining the quipu were known as "quipu camayocs" or "keeper of the quipu."

Since they had no written language and very few ancient quipu are left, we can only speculate what the quipu was actually used for. It's fortunate quipu are still used today, so we may be able to learn about the ancient ones by seeing how the modern ones are used. Combine this with oral traditions and it appears they were used to keep records on the number of things.

Another mystery which remains is, what base did the Inca use ? All their neighbors used a base 60, but it appears the Inca used base 10. Recent discoveries, as yet unsubstantiated, back this theory. For our purpose, we will assume it was base 10.

Making a quipu was easy. Thin strings were looped around a larger cord. Knots of colored thread or string were then tied around the thinner strings. Where the knots were placed indicated the value. The closer to the large cord a knot was placed, the greater its value. They way a knot was tied and the color used may be significant, but without a written language, we just don't know.

Some quipu found were several feet in length, so it was very important for the quipu camayocs to remember the who, where and what of each string and its placement on the larger cord

Contributed by Steven Tuck

References.

McIntyre, Loren. The Lost Empire of the Incas, National Geographic, Dec. 1973, 729 - 766.

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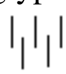
Fractions and Ancient Egypt

Ancient Egyptians had an understanding of fractions, however they did not write simple fractions as 3/5 or 4/9 because of restrictions in notation. The Egyptian scribe wrote fractions with the numerator of 1. They used the hieroglyph  "an open mouth" above the number to indicate its reciprocal. The number 5, written  , as a fraction 1/5 would be written  .

There are some exceptions. There was a special hieroglyph for 2/3, , and some evidence that 3/4 also had a special hieroglyph. All other fractions were written as the sum of unit fractions. For example 3/8 was written as 1/4 + 1/8.

The Egyptians had a need for fractions, such as the division of food, supplies, either equally or in a specific ratio. For example a division of 3 loaves among 5 men would require the fraction of 3/5. As new situations arose the Egyptians developed special techniques for dealing with the notation they already had, which meant the fraction was expressed as a sum of the unit fraction. Today as new concepts arise, mathematicians devise n new notation to deal with the situation.

Fractions were so important to the Egyptians that of the 87 problems in the Rhind Mathematical Papyrus only six did not involve fractions. Because the Egyptians performed their multiplications and divisions by doubling and halving, it was necessary to be able to double fractions. The scribes would create tables with calculations of fractions along with integers. These tables would be used as references so that temple personnel could carry out the fractional divisions on the food and supplies.

Contributed by Audrey Smalley

References.

Gillings, Richard J. Mathematics in the Time of the Pharaohs. (1982), Dover.

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The Mayan Number System

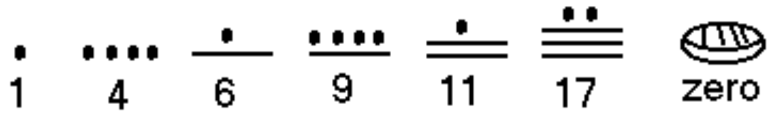
The Mayan number system dates back to the fourth century and was approximately 1,000 years more advanced than the Europeans of that time. This system is unique to our current decimal system, which has a base 10, in that the Mayan's used a vigesimal system, which had a base 20. This system is believed to have been used because, since the Mayan's lived in such a warm climate and there was rarely a need to wear shoes, 20 was the total number of fingers and toes, thus making the system workable. Therefore two important markers in this system are 20, which relates to the fingers and toes, and five, which relates to the number of digits on one hand or foot.

The Mayan system used a combination of two symbols. A dot (.) was used to represent the units (one through four) and a dash (-) was used to represent five. It is thought that the Mayan's may have used an abacus because of the use of their symbols and, therefore, there may be a connection between the Japanese and certain American tribes (Ortenzi, 1964). The Mayan's wrote their numbers vertically as opposed to horizontally with the lowest denomination on the bottom. Their system was set up so that the first five place values were based on the multiples of 20. They were 1 (20^0), 20 (20^1), 400 (20^2), 8,000 (20^3), and 160,000 (20^4). In the Arabic form we use the place values of 1, 10, 100, 1,000, and 10,000. For example, the number 241,083 would be figured out and written as follows:

Mayan Numbers	Place Value	Decimal Value
	1 times 160,000	= 160,000
	10 times 8,000	= 80,000
	2 times 400	= 800
	14 times 20	= 80
	3 times 1	= 3

This number written in Arabic would be 1.10.2.14.3 (McLeish, 1991, p. 129).

The Mayan's were also the first to symbolize the concept of nothing (or zero). The most common symbol was that of a shell ( ) but there were several other symbols (e.g. a head). It is interesting to learn that with all of the great mathematicians and scientists that were around in ancient Greece and Rome, it was the Mayan Indians who independently came up with this symbol which usually meant completion as opposed to zero or nothing. Below is a visual of different numbers and how they would have been written:



In the table below are represented some Mayan numbers. The left column gives the decimal equivalent for each position of teh Mayan number. Remember the numbers are read from bottom to top. Below each Mayan number is its decimal equivalent.

8,000						•••
400			•	•	••	• — — —
20	•	••	••	—	•• —	
units			—	••• —	••• — —	•••• — —
	20	40	445	508	953	30,414

It has been suggested that counters may have been used, such as grain or pebbles, to represent the units and a short stick or bean pod to represent the fives. Through this system the bars and dots could be easily added together as opposed to such number systems as the Romans but, unfortunately, nothing of this form of notation has remained except the number system that relates to the Mayan calendar.

For further study: The 360 day calendar also came from the Mayan's who actually used base 18 when dealing with the calendar. Each month contained 20 days with 18 months to a year. This left five days at the end of the year which was a month in itself that was filled with danger and bad luck. In this way, the Mayans had invented the 365 day calendar which revolved around the solar system.

Contributed by Mikelle Mercer

References.

1. McLeish, J. (1991). The story of numbers. New York, NY: Fawcett Columbine.
2. Ortenzi, E. C. (1964). Numbers in ancient times. Portland, ME: J. Weston Walch.
3. Roys, R. L. (1972). The Indian background of colonial Yucatan. Norman, OK: University of Oklahoma Press.
4. Thompson, J. E. S. (1967). The rise and fall of Maya civilization. Norman, OK: University of Oklahoma Press.
5. Trout, L. (1991). The Maya. New York, NY: Chelsea House Publishers.

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The Egyptian Number System

How do we know what the Egyptian language of numbers is? It has been found on the writings on the stones of monument walls of ancient time. Numbers have also been found on pottery, limestone plaques, and on the fragile fibers of the papyrus. The language is composed of heiroglyphs, pictorial signs that represent people, animals, plants, and numbers.

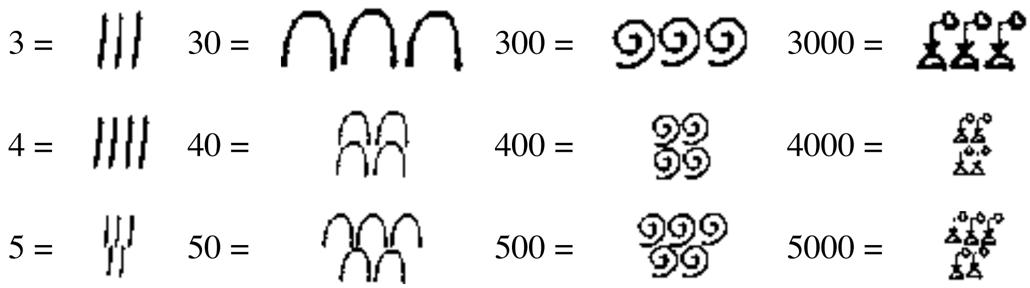
The Egyptians used a written numeration that was changed into hieroglyphic writing, which enabled them to note whole numbers to 1,000,000 . It had a decimal base and allowed for the additive principle. In this notation there was a special sign for every power of ten. For I, a vertical line; for 10, a sign with the shape of an upside down U; for 100, a spiral rope; for 1000, a lotus blossom; for 10,000 , a raised finger, slightly bent; for 100,000 , a tadpole; and for 1,000,000, a kneeling genie with upraised arms.

Decimal Number	Egyptian Symbol	
1 =		staff
10 =		heel bone
100 =		coil of rope
1000 =		lotus flower
10,000 =		pointing finger
100,000 =		tadpole
1,000,000 =		astonished man

This hieroglyphic numeration was a written version of a concrete counting system using material objects. To represent a number, the sign for each decimal order was repeated as many times as necessary. To make it easier to read the repeated signs they were placed in groups of two, three, or four and arranged vertically.

Example 1.

1 =		10 =		100 =		1000 =	
2 =		20 =		200 =		2000 =	



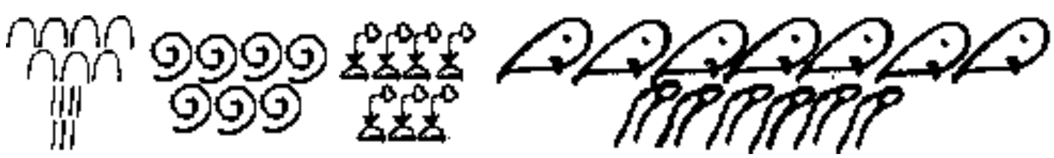
In writing the numbers , the largest decimal order would be written first. The numbers were written from right to left.

Example 2.

46,206 = 

Below are some examples from tomb inscriptions.

A B C D



77 700 7000 760,00

Addition and Subtraction

The techniques used by the Egyptians for these are essentially the same as those used by modern mathematicians today.The Egyptians added by combining symbols. They would combine all the units ( ) together, then all of the tens ( ) together, then all of the hundreds ( ), etc. If the scribe had more than ten units ( ), he would replace those ten units by . He would continue to do this until the number of units left was les than ten. This process was continued for the tens, replacing ten tens with , etc.

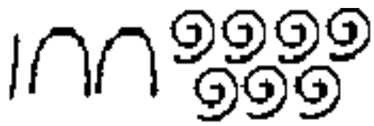
For example, if the scribe wanted to add 456 and 265, his problem would look like this

 (= 456)

 (= 265)

The scribe would then combine all like symbols to get something like the following

He would then replace the eleven units () with a unit () and a ten (). He would then have one unit and twelve tens. The twelve tens would be replaced by two tens and one one-hundred. When he was finished he would have 721, which he would write as

.

Subtraction was done much the same way as we do it except that when one has to borrow, it is done with writing ten symbols instead of a single one.

Multiplication

Egyptians method of multiplication is fairly clever, but can take longer than the modern day method. This is how they would have multiplied 5 by 29

*1	29
2	58
*4	116
1 + 4 = 5	29 + 116 = 145

When multiplying they would began with the number they were multiplying by 29 and double it for each line. Then they went back and picked out the numbers in the first column that added up to the first number (5). They used the distributive property of multiplication over addition.

$$29(5) = 29(1 + 4) = 29 + 116 = 145$$

Division

The way they did division was similar to their multiplication. For the problem 98/7 , they thought of this problem as 7 times some number equals 98. Again the problem was worked in columns.

1	7
2	*14
4	*28
8	*56

2 + 4 + 8 = 14   14 + 28 + 56 = 98

This time the the numbers in the right-hand column are marked which sum to 98 then the corresponding numbers in the left-hand column are summed to get the quotient.
So the answer is 14. 98 = 14 + 28 + 56 = 7(2 + 4 + 8) = 7*14

Contributed by Lloyd Holt

References:

1. Boyer, Carl B. - A History of Mathematics, John Wiley, New York 1968
2. Gillings, Richard J. - Mathematics in the Time of the Pharaohs, Dover, New York, 1982
3. Jason Gilman, David Slavit, - Ancient Egyptian Mathematics., Washington State University, 1995

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The Greek Number System

The Greek numbering system was uniquely based upon their alphabet. The Greek alphabet came from the Phoenicians around 900 B.C. When the Phoenicians invented the alphabet, it contained about 600 symbols. Those symbols took up too much room, so they eventually narrowed it down to 22 symbols. The Greeks borrowed some of the symbols and made up some of their own. But the Greeks were the first people to have separate symbols, or letters, to represent vowel sounds. Our own word "alphabet" comes from the first two letters, or numbers of the Greek alphabet -- "alpha" and "beta." Using the letters of their alphabet enabled them to use these symbols in a more condensed version of their old system, called Attic. The Attic system was similar to other forms of numbering systems of that era. It was based on symbols lined up in rows and took up a lot of space to write. This might not be to bad, except that they were still carving into stone tablets, and the symbols of the alphabet allowed them to stamp values on coins in a smaller, more condensed version.

Attic symbols

𐅅	=	500
Η	=	100
Δ	=	10
Γ	=	5

Ι = 1

For example, 𐅅ΗΗΗΔΔΔΔΓΙΙΙΙ represented the number 849

The original Greek alphabet consisted of 27 letters and was written from the left to the right. These 27 letters make up the main 27 symbols used in their numbering system. Later special symbols, which were used only for mathematics vau, koppa, and sampi, became extinct. The New Greek alphabet nowadays uses only 24 letters.

1	α	alpha	10	ι	iota	100	ρ	rho
2	β	beta	20	κ	kappa	200	σ	sigma
3	γ	gamma	30	λ	lambda	300	τ	tau
4	δ	delta	40	μ	mu	400	υ	upsilon
5	ϵ	epsilon	50	ν	nu	500	ϕ	phi
6	ϛ	vau*	60	ξ	xi	600	χ	chi
7	ζ	zeta	70	ο	omicron	700	ψ	psi
8	η	eta	80	π	pi	800	ω	omega
9	θ	theta	90	ϙ	koppa*	900	ϡ	sampi

*vau, koppa, and sampi are obsolete characters

If you notice, the Greeks did not have a symbol for zero. They could string these 27 symbols together to represent any number up to 1000. By putting a comma in front of any symbol in the first row, they could now write any number up to 10,000.

Here are representations for 1000, 2000 and the number we gave above 849.

,α = 1000 ,β = 2000 etc. ωμθ = 849

This works great for smaller numbers, but what about larger numbers? Here the Greeks went back to the Attic System, and used the symbol M for 10,000. And used multiples of 10,000 by putting symbols above M.

Mωμθ = 10,849

,ζρoε (above M)
M ,εωoε = 71,755,875

Contributed by Erik Sorum

References:

Burton, David M. The History of Mathematics - An Introduction. Dubuque, Iowa: William C. Brown, 1988.

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The Babylonian Number System

The Babylonians lived in Mesopotamia, which is between the Tigris and Euphrates rivers. They began a numbering system about 5,000 years ago. It is one of the oldest numbering systems. The first mathematics can be traced to the ancient country of Babylon, during the third millennium B.C. Tables were the Babylonians most outstanding accomplishment which helped them in calculating problems.

One of the Babylonian tablets, Plimpton 322, which is dated from between 1900 and 1600 BC, contains tables of Pythagorean triples for the equation $a^2 + b^2 = c^2$. It is currently in a British museum.

Nabu - rimanni and Kidinu are two of the only known mathematicians from Babylonia. However, not much is known about them. Historians believe Nabu - rimanni lived around 490 BC and Kidinu lived around 480 BC.

The Babylonian number system began with tally marks just as most of the ancient math systems did. The Babylonians developed a form of writing based on cuneiform. Cuneiform means "wedge shape" in Latin. They wrote these symbols on wet clay tablets which were baked in the hot sun. Many thousands of these tablets are still around today. The Babylonians used a stylist to imprint the symbols on the clay since curved lines could not be drawn.

The Babylonians had a very advanced number system even for today's standards. It was a base 60 system (sexigesimal) rather than a base ten (decimal). Base ten is what we use today.

The Babylonians divided the day into twenty-four hours, each hour into sixty minutes, and each minute to sixty seconds. This form of counting has survived for four thousand years.

Any number less than 10 had a wedge that pointed down.

Example: 4 

The number 10 was symbolized by a wedge pointing to the left.

Example: 20 

Numbers less than 60 were made by combining the symbols of 1and 10.

Example: 47 

As with our numbering system, the Babylonian numbering system utilized units, ie tens, hundreds, thousands.

Example: 64 

However, they did not have a symbol for zero, but they did use the idea of zero. When they wanted to express zero, they just left a blank space in the number they were writing.

When they wrote "60", they would put a single wedge mark in the second place of the numeral.

When they wrote "120", they would put two wedge marks in the second place.

Following are some examples of larger numbers.

Example: 79883

$(22*602^2)+(11*60)+23$

Example: 5220062

$(24*60^3) + (10*60^2) + (1*60) + 2$

Contributed by Jeremy Troutman

References:

1. URL:http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Babylonian_and_Egyptian.html 6-12-00 6:00 pm
2. URL:http://www.angelfire.com/il2/babylonianmath/mathematicians.html 6-12-00 6:00 pm
3. Boyer, Merzbach. A History of Mathematics. John Wiley & Sons, 1989. Second Edition.

4. Bunt, Jones, and Bedient. The Historical Roots of Elementary Mathematics. Dover Publications. 1988.

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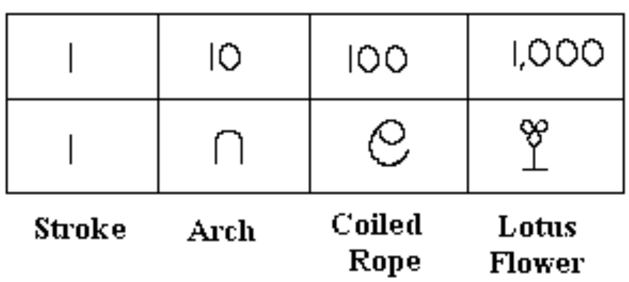
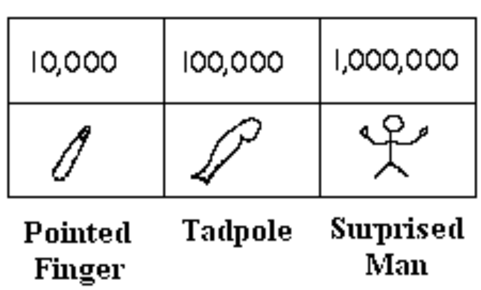
Where Did Numbers Originate?

Thousands of years ago there were no numbers to represent �two� or �three�. Instead fingers, rocks, sticks or eyes were used to represent numbers. There were neither clocks nor calendars to help keep track of time. The sun and moon were used to distinguish between 1 PM and 4 PM. Most civilizations did not have words for numbers larger than two so they had to use terminology familiar to them such as �flocks� of sheep, �heaps� of grain, or �lots� of people. There was little need for a numeric system until groups of people formed clans, villages and settlements and began a system of bartering and trade that in turn created a demand for currency. How would you distinguish between five and fifty if you could only use the above terminology?

Paper and pencils were not available to transcribe numbers. Other methods were invented for means of communication and teaching of numerical systems. Babylonians stamped numbers in clay by using a stick and depressing it into the clay at different angles or pressures and the Egyptians painted on pottery and cut numbers into stone.

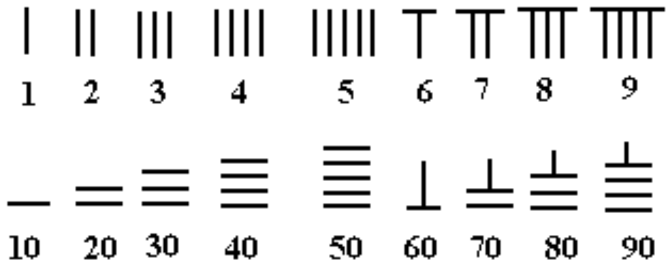
Numerical systems devised of symbols were used instead of numbers. For example, the Egyptians used the following numerical symbols:





From Esther Ortenzi, Numbers in Ancient Times. Maine: J. Weston Walch, 1964, page 9.

The Chinese had one of the oldest systems of numerals that were based on sticks laid on tables to represent calculations. It is as follows:



From David Smith and Jekuthiel Ginsburg, Numbers and Numerals.
W. D. Reeve, 1937, page 11.

From about 450 BC the Greeks had several ways to write their numbers, the most common way was to use the first ten letters in their alphabet to represent the first ten numbers. To distinguish between numbers and letters they often placed a mark (/ or �) by each letter:

A′ B′ Γ′ Δ′ E′ F′ Z′ H′ Θ′

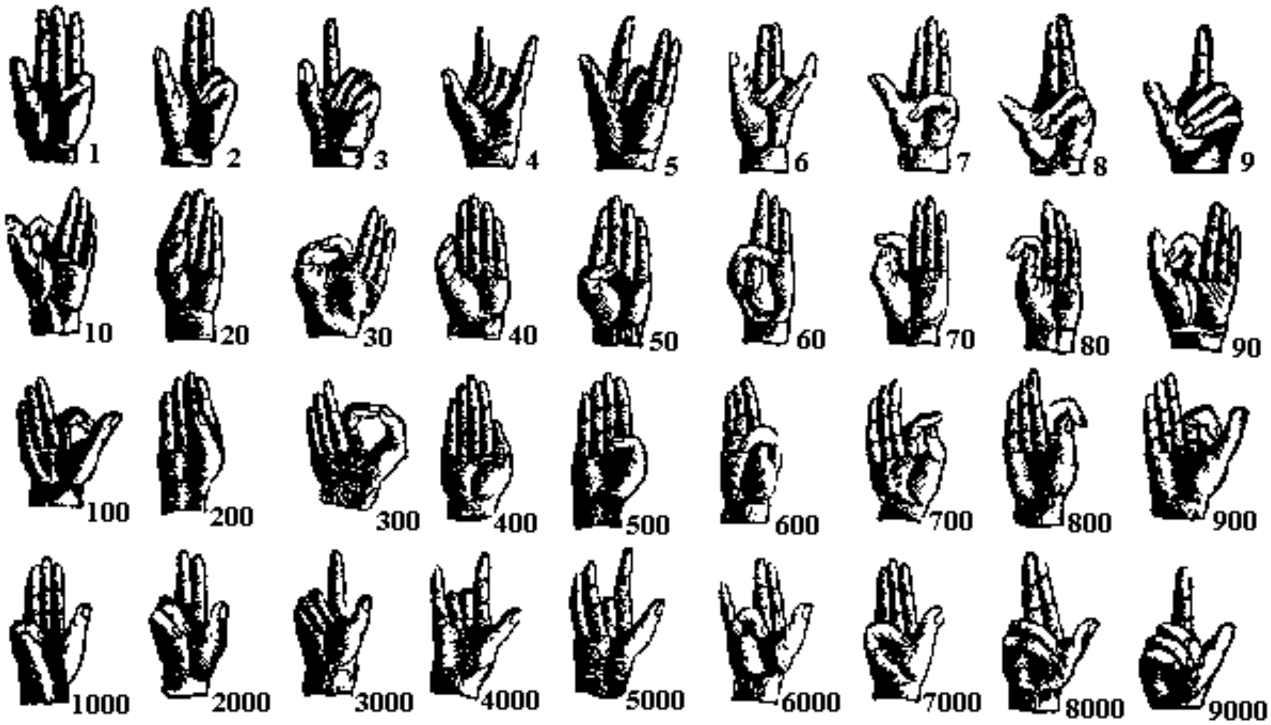
From David Smith and Jekuthiel Ginsburg, Numbers and Numerals.
W. D. Reeve, 1937, page 12.

The Roman numerical system is still used today although the symbols have changed from time to time. The Romans often wrote four as IIII instead of IV, I from V. Today the Roman numerals are used to represent numerical chapters of books or for the main divisions of outlines. The earliest forms of Roman numeral values are:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

From David Smith and Jekuthiel Ginsburg, Numbers and Numerals.
W. D. Reeve, 1937, page 14.

Finger numerals were used by the ancient Greeks, Romans, Europeans of the Middle Ages, and later the Asiatics. Still today you can see children learning to count on our own finger numerical system. The old system is as follows:



FINGER SYMBOLS
(From a manual published in 1520)

From Tobias Dantzig, Number: The Language of Science.
Macmillan Company, 1954, page 2.

From counting by means of �flocks� to finger symbols our current numerical system has evolved from the Hindu numerals to present day numbers. The journey has taken us from 2400 BC to present day and we still use some of the old numerical systems and symbols. Our system of numerics is ever changing and who knows what it will look like in 2140 AD. Will we still count using our fingers or will mankind invent a new numerical tool?

Sanscrit letters of the 11. Century A.D.		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Apices of Boethius and of the Middle Ages	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8	9	[illegible]
Gubar-numerals of the West Arabs	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	o
Numerals of the East Arabs	1	[illegible]	[illegible]	[illegible] or [illegible]	o or β	[illegible]	V	Λ	9	•
Numerals of Maximus Planudes.	1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	v	Λ	9	o
Devangari-numerals.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6	[illegible]	[illegible]	o
From the *Mirror of the World*, printed by Caxton, 1480	1	2	3	4	[illegible]	6	[illegible]	8	9	0
From the Bamberg Arithmetic by Wagner, 1488.	1	z	3 or 3	[illegible] or 4	[illegible] or 5	6	Λ or 7	8	9	0
From *De Arts Supp-urtandi* by Tonstall, 1522	1	2	3	4	5	6	7	8	9	10
This chart shows the change of numbers from their ancient to their present-day forms.										

This Chart was reconstructed from Esther Ortenzi, Numbers in Ancient Times.
Maine: J. Weston Walch, 1964, page 23.

Contributed by Carey Eskridge Lybarger

References:

1. David E. Smith and Jekuthiel Ginsburg. Numbers and Numerals. W. D. Reeves, 1937
2. Esther C. Ortenzi. Numbers in Ancient Times. J. Weston Walsh, 1964.
3. Tobias Dantzig. Number: The Language of Science. Macmillan Company, 1954.

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