Why study number theory?

(adapted from *Elementary Number Theory* by U. Dudley)

- Because professor says you must.
- Because you won't graduate if you don't.
- Because you have to take *something*.
- Because it gives your mind valuable training in thinking logically.
- Because numbers might be interesting.
- Because numbers are a fundamental part of a person's mental universe and hence worth looking into.
- Because some of the most powerful human minds that ever existed were concerned with numbers, and what powerful minds study is worth studying.
- Because you want to know everything about numbers: what makes them work and what they do.
- Because mathematics contains some beautiful things, and someone told you that number theory contained some of the *most* beautiful – and few of the most ugly – things.
- Because it is fun.
- Because there are still many easy-to-state unsolved problems in number theory, and that's cool (and driving a lot of powerful minds crazy). Turn over for some samples.

Some unsolved problems in number theory

(1) (Goldbach Conjecture) Can all positive even integers greater than 4 be expressed as the sum of two primes?

(2) (Twin Prime Conjecture) Are there infinitely many twin primes, namely pairs of prime numbers of the form $(p, p+2)$?

(3) ($3n+1$ Problem) Start with any integer n. Obtain a new integer m by halving n if it is even or taking $3n+1$ if it is odd. If m is even, take half of that. If it is odd, take $3m+1$. Keep doing this. Is it true that this iterative procedure always ends in 1?

(4) (Catalan's Problem) Are 8 and 9 the only two consecutive powers? I.e. are there numbers x and y, not 2 and 3, and primes p and q, such that $y^p - x^q = 1$?

(5) (Palindrome Conjecture) Pick an integer. Reverse its digits and add the resulting integer to the original one. If the result isn't a palindrome (its digits don't read the same forward and backward), repeat the process. Do all integers eventually become palindromes through this process?

(6) Are there any odd perfect numbers, namely odd numbers which are the sum of their divisors?

(7) Are there infinitely many Mersenne primes, namely primes of the form $2^p - 1$, where p is a prime?

(8) Are there infinitely many Fermat primes, namely primes of the form $2^{2^n} + 1$, where n is a positive integer?

(9) Are there infinitely many primes of the form $n^2 + 1$, where n is a positive integer?

(10) Are there infinitely many primes whose digits are all 1?

(11) Does there always exist a prime between n^2 and $n^2 + n$, for n a positive integer?

(12) (Riemann Hypothesis) Well, this one is hard to state, but you'll get $1,000,000 if you solve it!