APMA 213, Summer 2005
Final Exam

Name: ______________________________ Student ID: ___________________________

Please sign the pledge: On my honor as a student I have neither given nor received aid on this examination.

Signature: ____________________________________

Directions: Check that your test has 11 pages, including this one and the blank one on the bottom. The next to last page should be a table of Laplace transforms. Please answer all questions and show your work. **Write neatly: solutions deemed illegible will not be graded, so no credit will be given.** Closed book, closed notes, no calculators.

1. (16 points) ______________
2. (5 points) _______________
3. (10 points) ______________
4. (10 points) ______________
5. (10 points) ______________
6. (4 points) _______________
7. (10 points) ______________
8. (5 points) _______________

Total (out of 70): ______________

1. (2 pts each) Answer each of the following questions by circling TRUE or FALSE. You do not need to justify you answer and no partial credit will be given.

(b) TRUE FALSE

An equation of the form $M(x) + N(y)y' = 0$ is always separable.

(b) TRUE FALSE

Equation $y'' + p(t)y' + q(t)y = g(t)$ with initial conditions $y(t_0) = y_0$, $y'(t_0) = y'_0$, always has a solution.

(c) TRUE FALSE

The method of variation of parameters is a method for finding a particular solution to a non-homogeneous linear differential equation.

(d) TRUE FALSE

The value of the Dirac delta function $\delta(t)$ at $t = 0$ is 1.

(e) TRUE FALSE

If y_1 and y_2 are solutions of the equation $y'' + p(t)y' + q(t)y = 0$, then their Wronskian is $y_1 y'_2 - y'_1 y_2$.

(f) TRUE FALSE

Matrix $\begin{pmatrix} 1 & 1-i \\ 1+i & 2 \end{pmatrix}$ is its own adjoint.

(g) TRUE FALSE

Every system of first order linear equations can be turned into a single linear first order equation.

(h) TRUE FALSE

If $\mathbf{A}$ is a nonsingular matrix, then the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ has a unique solution.

2. (5 pts) Find a general solution to the equation

$$y' + 2y = te^{-2t}.$$

3. (10 pts) Use the method of variation of parameters to find a general solution to the equation

$$y'' + y = \sec t.$$

4. Find the Laplace transform of the following functions.

(a) (5 pts) $f(t) = \begin{cases} 0, & 0 \le t < \pi, \\ \sin t, & \pi \le t < 2\pi, \\ 0, & t \ge 2\pi. \end{cases}$

(b) (5 pts) $f(t) = \delta(t - \frac{\pi}{2}) \sin(3t)$.

5. (10 pts) Use Laplace transforms to find the solution of the initial value problem

$$y'' - 4y' + 4y = 0, \qquad y(0) = 1, \quad y'(0) = 1.$$

6. (4 pts) Rewrite the third order equation $4y''' + y' = e^t$ as a system of first order equations.

7. (10 pts) Find the particular solution to the initial value problem

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

8. (5 pts) Given that $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$ has a repeated eigenvalue $\lambda = 2$ with an eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, we know that one solution to the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ is $\mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t}$. Find another solution, $\mathbf{x}^{(2)}$, which is linearly independent from $\mathbf{x}^{(1)}$.

Elementary Laplace Transforms

$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1	$\frac{1}{s}, \quad s>0$
e^{at}	$\frac{1}{s-a}, \quad s>a$
$t^n, \quad$ n=positive integer	$\frac{n!}{s^{n+1}}, \quad s>0$
$\sin at$	$\frac{a}{s^2+a^2}, \quad s>0$
$\cos at$	$\frac{s}{s^2+a^2}, \quad s>0$
$\sinh at$	$\frac{a}{s^2-a^2}, \quad s>\lvert a\rvert$
$\cosh at$	$\frac{s}{s^2-a^2}, \quad s>\lvert a\rvert$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s>a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s>a$
$t^n e^{at}, \quad$ n=positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s>a$
$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s>0$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$e^{ct}f(t)$	$F(s-c)$
$f(ct)$	$\frac{1}{c}F(\frac{s}{c}), \quad c>0$
$\delta(t-c)$	e^{-cs}
$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$
$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$