Chapter Review Sheets for Boyce and DiPrima's
Elementary Differential Equations and Boundary Value Problems
7th Edition

Chapter 1: Introduction

Definitions:

Differential Equation
Mathematical Model
Direction (Slope) Field
Equilibrium Solution
Initial Condition, Initial Value Problem (IVP)
General Solution
Ordinary Differential Equation (ODE), Partial Differential Equation (PDE)
Order, Linear, Nonlinear, Linearization

Important Skills:

Derive differential equations that mathematically model simple problems. (Example 1, p.2; Also see p.7)
Construct a direction field for a first order ODE, and sketch approximate solutions. (Example 2, p.3)
Graph the integral curves of a general solution (Example 2, p.13)
Know what an initial value problem is, and how to show a given function is a solution to one. (Example 2, p.13)
Know the difference between an ordinary differential equation and partial differential equation. (p.17)
Know how to classify differential equations as order, and linearity. (p.18 & 19)

Chapter 2: First Order Differential Equations

Definitions:

First Order Ordinary Differential Equation
Integrating Factor, Integral Curves
Separable
Existence and Uniqueness of Solutions
General Solutions, Implicit Solutions
Autonomous, Logistic Growth, Equilibrium Solutions, Critical Points
Exact ODE
Tangent Line Method (Euler's Method)
First Order Difference Equation

Theorems:

Theorem 2.4.1: Existence and uniqueness of solutions to linear first order ODE's.
Theorem 2.4.2: Existence and uniqueness of solutions to first order ODE's $y' = f(t, y) \quad y(t_0) = y_0$.
Theorem 2.6.1: Existence and uniqueness of solutions to exact first order ODE's.
Theorem 2.8.1: Restatement and elaboration of theorem 2.4.2.

Important Skills:

Be able to determine if a first order differential equation is linear or nonlinear. Equation (3) on page 30 gives the form for a linear ODE.
If the differential equation is linear, compute the integrating factor, and then the general solution. (Example 4, p. 36)
Be able to graph integral curves for an ODE. (Example 4, p. 36)
If it's nonlinear, is it separable? If it's separable, you will need to compute two different integrals. It crucial to know integration of basic functions and integral methods from your calculus course. For example, various substitutions, integration by parts, and partial fractions will all be utilized. (Examples 2&3, p. 42 & 44)
If the differential equation is not separable, is it exact? If so, solve it using the method in section 2.6 (Example 2, p. 92)
If it isn't separable or exact, check for substitutions that would convert it into a linear equation, or a nonlinear equation that is then separable. For example, exercises 27-31 in section 2.4, show how Bernoulli equations can be transformed into linear equations.
Know how to obtain approximate solutions using Euler's method if an analytical solution cannot be found. (Example 2, p.100)
Understand the three steps in the process of mathematical modeling. (Example 3 p. 54)
Determine the existence and uniqueness of solutions to differential equations. (Example 2, p. 66)
Know how to recognize autonomous equations, and utilize the direction field to represent solutions to them. Be able to determine asymptotically stable, semi-stable, and unstable equilibrium solutions. (Example 1, p. 80)

Relevant Applications:

Mixing Problems, Compound Interest, Motion in a Gravitational Field, Radioactive Carbon Dating

Chapter 3: Second Order Linear Equations

Definitions:

Homogeneous, Nonhomogeneous
Characteristic Equation
Wronskian
General Solution, Fundamental Set of Solutions
Linear Independence
Particular Solution
Period, Natural Frequency, Amplitude, Phase
Overdamped, Critically Damped, Underdamped
Resonance
Transient Solution, Steady-State Solution or Forced Response

Theorems:

Theorem 3.2.1: Existence and uniqueness of solutions to $y'' + p(t)\, y' + q(t) y = g(t)$, $y(t_0) = y_0 \;\; y'(t_0) = y_0'$.

Theorem 3.2.2: Principle of superposition. If y_1 and y_2 are solutions to $y'' + p(t)\, y' + q(t) y = 0$, so is $c_1 y_1 + c_2 y_2$ for any constants c_1 and c_2.

Theorem 3.2.3: Finding solutions to Eq. (2) an Eq. (3), using the Wronskian at the initial conditions.

Theorem 3.2.4: Representing general solutions to second order linear homogeneous ODE's

Theorem 3.2.5: Existence of a fundamental set of solutions.

Theorem 3.3.1: Linear independence of functions and the Wronskian.

Theorem 3.3.2: Abel's Theorem.

Theorem 3.3.3: Linear independence of solutions to $y'' + p(t)\, y' + q(t) y = g(t)$ and the Wronskian.

Theorem 3.6.1: Relating differences in nonhomogeneous solutions to fundamental solutions. (Used to prove the following theorem.)

Theorem 3.6.2: General solutions to linear nonhomogeneous ODE's.

Theorem 3.7.1: General solutions to linear nonhomogeneous ODE's. (Using variation of parameters to determine the particular solution.)

Important Skills:

Be able to determine if a second order differential equation is linear or nonlinear, homogeneous or nonhomogeneous. (If it can be put into the form given by Equation (3) in page 130, it is linear.) Most of the chapter deals with linear equations.

Important exceptions are two methods given in Section 3.1 exercises 28-33, which show how to solve second order differential equations missing the dependent variable, and Exercises 34-39, which shows how to solve equations missing the independent variable.

Can you recognize a homogeneous equation with constant coefficients, and derive the characteristic equation? (Example 3, p.143) This equation will be quadratic, so know the quadratic formula, and the types of solutions one gets; real and distinct, repeated, and complex conjugate. These three cases will crucial to the types of solutions one gets to constant coefficient homogeneous differential equations.

Be able to write down fundamental solution sets to homogeneous equations. This means finding two linearly independent solutions. You can use the Wronskian to show if two solutions are linearly independent. (Example 3, page 141)

Reduction of order is a way to take a know solution and produce a second linearly independent one. Know it! (Example 3, p.166)

What are the fundamental solution sets for each of the three case of roots when solving constant coefficient equations? The summary is on p.165. (Example 3, p.134; Example 2, p.163; Example 2 p.156)
Solutions to second order nonhomogeneous equations have two components. There is the homogeneous solution, and particular, or nonhomogeneous solution. (Theorem 3.6.2 p.170)
To find particular solutions you must know the method of undetermined coefficients, and variation of parameters. (Example 4, p. 173; Example 1, p. 180)
Mechanical vibrations give excellent examples for utilizing all the techniques in the chapter. Know the difference between damped and undamped vibrations, forced and unforced situations.
For the unforced case, if there is no dampening, the motion is sinusoidal. Be able to determine the natural spring frequency. (Example 2, p.191) If there is dampening, know the three different cases; underdamped, critically damped, and overdamped, depending on roots to the characteristic equation. If underdamped, know the quasi period. (Example 3, p.191) Know how to graph solutions in the three different cases of dampening.
For the forced problem, the cases separate into damped or undamped. If undamped, there is the possibility of resonance if the nonhomogeneous forcing term is sinusoidal with frequency equivalent to the natural spring frequency. (p.202)
If there is no resonance, then there will be beats. (p.201) Know how to derive and graph solutions in this case. You may need to brush up on some trigonometric identities.
For the damped case, know how to identify and graph transient and steady state solutions. (p.203)

Relevant Applications:

Mechanical Vibrations, Electric Circuits

Chapter 4: Higher Order Linear Equations

Definitions:

nth Order Linear ODE
Fundamental Set of Solutions, General Solution
Linear Dependence and Independence
Characteristic Polynomial, Characteristic Equation

Theorems:

Theorem 4.1.1: Existence and uniqueness of solutions to higher order linear ODE's.
Theorem 4.1.2: General solutions to higher order linear ODE's and the fundamental set of solutions

Important Skills:

The methods for solving higher order linear differential equations are extremely similar to those in the last chapter. There is simply n times the fun! The general solution to an nth order homogeneous linear differential equation is obtained by linearly combining n linearly independent solutions. (Equation 5, p.210)
The generalization of the Wronskian is given on page 211. It is used as in the last chapter to show the linear independence of functions, and in particular homogeneous solutions.
For the situation where there are constant coefficients, you should be able to derive the characteristic polynomial, and the characteristic equation, in this case each of nth order. Depending upon the types of roots you get to this equation, you will have solution sets containing functions similar to those in the second order case. (Examples 2-4, p.217-219)
The general solution of the nonhomogeneous problem easily extends to the nth order case. (Equation 9, p.212)
Both variation of parameters, and the method of undetermined coefficients generalize to determine particular solutions in the higher dimensional situation. (Example 3, p. 223; Example 1, p. 228)

Relevant Applications:

Double spring mass systems

Chapter 5: Series Solutions of Second Order Equations

Definitions:

Radius of Convergence, Interval of Convergence
Analytic
Recurrence Relation
Ordinary Point, Singular Point
Regular and Irregular Singular Points
Euler Equation, Indicial Equation
Exponents of Singularity
Bessel Equation

Theorems:

Theorem 5.3.1:	Existence of series solutions to linear ODE's near ordinary points, and their convergence properties.
Theorem 5.5.1:	General solutions to Euler equations.
Theorem 5.7.1:	Series solutions near regular singular points.

Important Skills:

Review power series, how to shift the index of summation, (Example 3, p.235) and tests for convergence. (Example 2, p.233)
Know how to find the interval of convergence for a power series. (Example 2, p.233)
Be able to determine all ordinary and singular points for a differential equation. (p. 238)
For all singular points, be able to categorize as either regular or irregular; (Equations (6) and (7) on page 257 give the criteria for a regular singular point.)
For ordinary points, equation (3) on page 239 gives the form of the solution. Be able to derive the recursion relation, as in example 1. If the recursion relation can be solved, one obtains the two linearly independent solutions of the homogenous problem. (Example 1, p.239)
The method described in the second paragraph on page 244 can be used to find the first several terms in each of the linearly independent homogeneous solutions.
Be able to determine lower bounds on the radius of convergence of the series solutions. (Example 4, p.252)
Series solutions near regular singular points require the ability to solve Euler equations. Be able to recognize Euler equations, and know how to derive the characteristic equation. Know the general solutions for the three case of roots to the characteristic equation. (Theorem 5.5.1, Examples 2 & 3, p. 262 & 263)
The assumption for the form of the series solution near regular points is given by (7) on page 268. Substitution into the differential equations will yield an indicial equation, as well as, an recursion relation. The solutions to the indicial equation are those to the associated Euler problem. (Example 1, p.268)
In cases where the roots to the indicial equation are equal or differ by an integer, the method must be slightly modified to obtain solutions, or one can use reduction of order. (p. 276 &277)
Finally, Bessel equations give good examples of series solutions near regular singular points, and several examples are given in section 5.8.

Chapter 6: The Laplace Transformation

Definitions:

Integral Transforms, Kernel
Improper Integral
Piecewise Continuous
Exponential Order
Unit Step Function (Heaviside Function)
Unit Impulse Function, Delta Function
Convolution
Transfer Function, Impulse Response

Theorems:

Theorem 6.1.1:	Comparison Test for Improper Integrals
Theorem 6.1.2:	Existence of the Laplace Transform, $F(s)$
Theorem 6.2.1:	Laplace Transform of $f'(t)$
Corollary 6.2.2:	Laplace Transform of $f^{(n)}(t)$
Theorem 6.3.1:	Transform of the unit step function, $u_c(t)$, times a shifted function, $f(t-c)$
Theorem 6.3.2:	First Translation Theorem; Inverse Transforming $F(s-c)$
Theorem 6.6.1:	Second Translation Theorem; Convolution Result

Important Skills:

The Laplace transformation is defined through an improper integral. You must be comfortable evaluating them. Hence you should review this topic in any calculus book.
Be able to calculate the transform of all the basic functions, given in the table on page 304. (Examples 5 & 6, p.297)
Even more importantly, know how to compute inverse transform functions using manipulative and translation methods. You may need to use partial fractions, but you should have already reviewed this for chapter 2. (Example 1, p.305)
Know how to transform derivatives of functions and linear differential equations. (Corollary 6.6.2, p.300, Example 1 & 2, p.305)
Understand the unit function, $u_c(t)$, as well as, the unit impulse function, $\delta(t)$, and how to use them in transforming and inverse transforming functions. (Example 1, p.310; Example 1, p.327)
The process of using the Laplace transform method is as follows; Given a differential equation for $y(t)$, one transforms both sides of the equation. One will need to input the initial values when transforming derivatives. Derivatives with respect to *t* transform to polynomials in *s*. If the differential equation is linear, then the resulting equation is linear in $Y(s)$. You simply solve this equation for $Y(s)$, and then use all the methods available to inverse transform $Y(s)$, and recover $y(t)$. (Example 1, p.305 for continuous forcing; Example 1, p.317 for discontinuous forcing.)

Relevant Applications:

Mechanical and electrical problems with discontinuous forcing functions.

Chapter 7: Systems of First Order Linear Equations

Definitions:

Systems of ODE's
Linear vs Nonlinear Systems
Solution
Homogenous and Nonhomogeneous Systems
Matrix, Transpose, Conjugate, Adjoint, Determinant
Scalar (Inner) Product, Orthogonal
Nonsingular (Invertible) and Singular (Noninvertible)
Row Reduction (Gaussian Elimination)
Linear Systems, Homogeneous, Nonhomogeneous
Augmented Matrix
Linear Dependence and Independence
Eigenvalues, Eigenvectors, Generalized Eigenvectors
Normalization
Multiplicity m, Simple Multiplicity ($m = 1$)
Self-Adjoint (Hermitian)
General Solution, Fundamental Set of Solutions
Phase Plane, Phase Portrait
Node, Saddle Point, Spiral Point, Improper Node
Fundamental Matrix
Similarity Transformation, Diagonalizable

Theorems:

Theorem 7.1.1: Existence and Uniqueness of Solutions to Systems of First Order IVP's
Theorem 7.1.2: Existence and Uniqueness of Solutions to Systems of First Order Linear IVP's
Theorem 7.4.1: Principle of Superposition of Solutions of Linear Systems of ODE's
Theorem 7.4.2: Fundamental Solution Sets for Linear Systems of ODE's
Theorem 7.4.3: Solutions to Linear Systems of ODE's, and the Wronskian
Theorem 7.4.4: Existence of Fundamental Solution Sets

Important Skills:

Find the inverse of a matrix. (Example 2, p.353)
Find the solution to a set of linear algebraic equations. (Example 1, p.358)
Determine if a set of vectors are linearly independent. (Example 3, p.361)
Find the eigenvalues and eigenvectors of a matrix. (Example 5, p.364)
Sketch a direction field for a 2 x 2 system of linear ODE's. (Example 2, p.376)
Find the general solution of a system of linear ODE's.
- Distinct Eigenvalues (Example 3, p.379)
- Complex Eigenvalues (Example 1, p.385)
- Repeated Eigenvalues (Example 2, p.402)

Find the fundamental matrix for a system of linear ODE's. (Example 2, p.395)
Find the similarity transformation to diagonalize a matrix. (Example 3, p.398)
Use the method of undetermined coefficients to find the particular solution to a nonhomogeneous linear system of ODE's. (Example 2, p.413)
Use the method of variation of parameters to find the particular solution to a nonhomogeneous linear system of ODE's. (Example 3, p.416)

Relevant Applications:

Multiple Spring Mass Problems, Multiple Tank Mixture Problems