Math 223, Spring '09
Homework 1 Solutions

Please read Chapter 1 in the textbook and turn in the solutions to the problems below. They refer to the handout on open problems we talked about on the first day of class.

Optional readings (both can be found in the "Handouts" folder of our conference):

- *Number_Systems.pdf*;
- *BriefHistoryOfNT.pdf*
- *Mersenne Primes.pdf*

(1) Show that the "$3n+1$" algorithm terminates at 1 with 6 as the starting value.

Solution: The sequence of numbers obtained by applying the "$3n+1$" algorithm to 6 is

$$6, \quad 6/2 = 3, \quad 3 \cdot 3 + 1 = 10, \quad 10/2 = 5, \quad 5 \cdot 3 + 1 = 16, \quad 16/2 = 8, \quad 8/2 = 4, \quad 4/2 = 2, \quad 2/2 = 1.$$

(2) A *perfect number* is a number which is the sum of its divisors. For example, 1, 2, and 3 divide 6 and $6 = 1 + 2 + 3$. Find another perfect number.

Solution: The first perfect number after 6 turns out to be 28. The divisors of 28 are 1, 2, 4, 7, 14 and

$$28 = 1 + 2 + 4 + 7 + 14.$$

(3) Find three *Mersenne primes*, namely primes of the form $2^p - 1$ where p is a prime.

Solution: Taking $p = 2, 3$ and 5 works, since

$$2^2 - 1 = 3, \quad 2^3 - 1 = 7, \quad 2^5 - 1 = 31$$

are all prime.