Math 305, Quiz 1 Solutions
September 13, 2007

Name:______________________________

(1) (5 pts) For each $n \in \mathbb{Z}$, define a map $f_n \colon \mathbb{Z} \to \mathbb{Z}$ by $f_n(x) = nx$. For which values of n is f_n injective or surjective?

Solution: Given x_1, $x_2 \in \mathbb{Z}$, suppose $f_n(x_1) = f_n(x_2)$, i.e. $nx_1 = nx_2$. If $n \neq 0$, it follows that $x_1 = x_2$. Thus f_n is injective for all $n \neq 0$. For $n = 0$, $f_0(x) = 0 \cdot x = 0$ and this is clearly not injective since everything is mapped to 0.

For surjectivity, take $y \in \mathbb{Z}$. Then $f_n(x) = nx = y$ means that $x = y/n$ if $n \neq 0$. However, y/n is only an integer if $n = \pm 1$. Thus $f_n(x)$ is surjective when $n = \pm 1$ (and again clearly not surjective when $n = 0$).

(2) (5 pts) Suppose that $\beta \colon S \to T$, $\gamma \colon S \to T$ and $\alpha \colon T \to U$ are all maps. Prove that, if α is one-to-one and $\alpha \circ \beta = \alpha \circ \gamma$, then $\beta = \gamma$ (Note: Two maps are equal if they give the same value when evaluated on any element of the domain.).

Solution: For any $x \in S$, we have $(\alpha \circ \beta)(x) = (\alpha \circ \gamma)(x)$. In other words, we have $\alpha(\beta(x)) = \alpha(\gamma(x))$. Since α is one-to-one, we can conclude that $\beta(x) = \gamma(x)$. Since x is arbitrary, we conclude that $\beta = \gamma$.