Math 305, Quiz 2
September 20, 2007

Name:______________________________

(1) (5 pts) Suppose G is the set of all continuous functions $f\colon \mathbb{R} \to \mathbb{R}$ such that $\int_0^1 f(x)\,dx = 1$. Determine whether G is a group under the usual multiplication of real-valued functions.

Solution: Consider the functions $f, g\colon \mathbb{R} \to \mathbb{R}$ given by $f(x) = 2x$ and $g(x) = 3x^2$ for all $x \in \mathbb{R}$. Then $f, g \in G$. However the product $fg\colon \mathbb{R} \to \mathbb{R}$ is given by $(fg)(x) = 6x^3$ does not belong to G since $\int_0^1 6x^3\,dx = 3/2$. Hence G is not a group.

(2) (5 pts) Suppose that G is a group where every element is its own inverse. Prove that G is abelian.

Solution: Let $a, b \in G$. Then $a * b = (a * b)^{-1} = b^{-1} * a^{-1} = b * a$. Hence G is abelian.