

Math 305, Quiz 3 Solutions
September 27, 2007

Name: _____

- (1) (5 pts) Write $(1\ 4\ 5)(1\ 2\ 3\ 5)(1\ 3) \in S_5$ as a single cycle or a product of disjoint cycles and then decide if this element is an even or an odd permutation.

Solution: The product is

$$(1\ 4\ 5)(1\ 2\ 3\ 5)(1\ 3) = (1)(2\ 3)(4\ 5) = (2\ 3)(4\ 5).$$

Since this permutation is a product of two transpositions, it is even.

- (2) (5 pts) Let $SL_n(\mathbb{R})$ be the subset of $GL_n(\mathbb{R})$ consisting of $n \times n$ matrices A with $\det(A) = 1$. Prove that $SL_n(\mathbb{R})$ is a subgroup of $GL_n(\mathbb{R})$.

Solution: First note that $SL_n(\mathbb{R})$ is nonempty since $\det(I) = 1$ where I is the identity matrix. For $A, B \in SL_n(\mathbb{R})$, $\det(AB) = \det(A)\det(B) = 1 \cdot 1 = 1$, so $SL_n(\mathbb{R})$ is closed. Since $\det(A^{-1}) = 1/\det(A)$ and $\det(A) = 1$, we have $\det(A^{-1}) = 1$. Thus $SL_n(\mathbb{R})$ contains inverses, and is therefore a subgroup of $GL_n(\mathbb{R})$.