Math 305, Quiz 4 Solutions
October 4, 2007

Name:______________________________

(1) (5 pts) Which of the letters

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Which of these letters has a group of symmetries of order 2? (We consider the letters as having no thickness.)

Solution: We want those letters with only one motion which preserves it (rotation or refletion). This motion, when done twice, should be the identity. The letters with this property are

A B C D E K M N S T U V W Y Z

The reason is the following: The only motion for A, M, T, U, V, W, and Y is reflection across the vertical line going through the middle of the letter; the only motion for B, C, D, E, and K is reflection across the horizontal line going through the middle of the letter; the only motion for N, S, and Z is rotation through π around the middle of the letter. Letters H, I, O, and X have more than one nontrivial symmetry; and F, G, J, L, P, Q, and R have none.

(2) (5 pts) For $a \in \mathbb{Z}$, prove that a^2 is congruent to either 0 or 1 modulo 4.

Solution: If a is even, then $a = 2k$ for some $k \in \mathbb{Z}$. Thus $a^2 = 4k^2$ and we have $4k^2 \equiv 0 \pmod 4$ since 4 divides $4k^2$.

If a is odd, then $a = 2k+1$ for some $k \in \mathbb{Z}$. Thus $a^2 = 4k^2 + 4k + 1 = 4k(k+1) + 1$ and we have $4k(k+1) + 1 \equiv 1 \pmod 4$ since 4 divides $4k(k+1) + 1 - 1 = 4k(k+1)$.