Math 305, Quiz 5 Solutions
October 25, 2007

Name:______________________________

(1) (5 pts) Prove that, if $\theta\colon G \to H$ is an isomorphism and G contains a subgroup of order m, then H contains a subgroup of order m.

Solution: Let $A \subseteq G$ be a subgroup of order m. We claim that $\phi(A)$ is a subgroup of H of order m. Clearly since ϕ is a bijection, then $\phi(A)$ is a set of order m, and therefore it is nonempty. Let $h, k \in \phi(A)$. Then there are $a, b \in A$ such that $\phi(a) = h$ and $\phi(b) = k$. Then $hk^{-1} = \phi(a)\phi(b)^{-1} = \phi(ab^{-1}) \in \phi(A)$. Hence $\phi(A)$ is a subgroup of H of order m.

(2) (5 pts) Use the Fundamental Theorem of Finite Abelian Groups to give representatives of five distinct isomorphism classes of groups of order p^4 where p is a prime.

Solution: The five distinct representatives are

$$\mathbb{Z}_{p^4},\ \mathbb{Z}_{p^3} \times \mathbb{Z}_p,\ \mathbb{Z}_{p^2} \times \mathbb{Z}_{p^2},\ \mathbb{Z}_{p^2} \times \mathbb{Z}_p \times \mathbb{Z}_p,\ \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p.$$