Math 305, Quiz 7 Solutions
November 29, 2007

Name:_____

(1) (5 pts) Use the extended version of Sylow's Theorem to show that every group of order 6 has only one subgroup of order 3. Also give an example of a group of order 6 with more than one subgroup of order 2.

Solution: By Sylow's Theorem, n_3 is congruent to 1 mod 3 so it can be only 1 or 4. But n_3 also has to divide 2, so it must be 1. For the second part, S_3 is an example.

(2) (5 pts) Give an example of a finite ring which is not an integral domain and an example of a noncommutative ring with unit.

Solution: Any $\mathbb{Z}_n$ where n is composite; ring of matrices $M(n, \mathbb{Z})$ for any $n \geq 2$.