

Math 306 Topics in Algebra, Spring 2013
Homework 3, due Friday, February 22

- (1) (5 pts) Section 4.5, problem 4 (p. 146), but only for D_6 (which the book calls D_{12}).
- (2) (5 pts) Use Extended Sylow's Theorem to verify that, if $|G| = 24$, then $n_2 = 1$ or 3 and $n_3 = 1$ or 4 . Then find n_2 and n_3 for S_4 by finding all Sylow 2-subgroups and all Sylow 3-subgroups of S_4 .
- (3) (5 pts/part)
 - (a) Prove that there are no simple groups of order 20.
 - (b) Prove that a group of order 175 must have normal subgroups of orders 7 and 25.
- (4) (7 pts) Classify groups of order 21.
- (5) (5 pts) Recall that, in the proof of the classification of groups of order 12, we let H be a Sylow 2-subgroup of G and K be a Sylow 3-subgroup of G . Prove that at least one of these subgroups is normal.
- (6) (7 pts) Section 4.5, problem 13 (p. 147).