Math 306: Field extensions

Simple extension: An extension $L\colon K$ such that $L = K(\alpha)$ for some $\alpha \in L$.

Algebraic/transcendental extension: An extension $L\colon K$ whose every element is algebraic, i.e. every element in L is a root of some non-zero polynomial with coefficients in K. Otherwise transcendental.

Finite extension: An extension $L\colon K$ whose degree is finite (i.e. L is a finite-dimensional vector space over K)

Normal extension: An extension $L\colon K$ with the property that every irreducible polynomial over K with at least one zero in L splits in L (i.e. all its roots are in L).

Normal closure: A normal closure of a finite extension $L\colon K$ is the smallest normal extension of K containing L.

Splitting field: Subfield Σ of $\mathbb{C}$ is a splitting field for $f \in K[x]$ if $K \subset \Sigma$, f splits over Σ, and Σ is the smallest field over which f splits (the last condition is equivalent to saying the $\Sigma = K(\text{roots of } f)$).

Separable extension: An extension $L\colon K$ whose every element α is separable over K, i.e. α's minimal polynomial f (over K) is separable over K, i.e. f has no multiple zeros in its splitting field.

Galois extension: An extension that is normal and separable.