Life of Galois

Galois was born October 25, 1811, in Bourg-la-Riene, near Paris, France. His parents, Nicholas Gabriel and Adelaide Marie Galois, were both intelligent and well-respected. In fact, four years after Évariste's birth, his father was elected mayor of their town.

Galois' education began like that of most middle-class French children; until he was 12, his mother tutored him at home, along with his older sister and younger brother. Under her tutelage, Galois learned much about classical Latin and Greek, but almost nothing about mathematics, in which neither his mother nor his father had any background.

In 1823, at age 12, Galois entered the school of Louis-le-Grand in Paris. The boarding school, located in Paris, had been attended by Robespierre, Voltaire and Victor Hugo, but, seemed 'dedicated to mediocrity and misery. The students were wakened at five-thirty every morning to dress in dark, unheated dormitories that were crowded with bug-infested beds' classes began early in the morning and continued until evening, with an undue emphasis on Latin and the classics'.

At the school, mathematics was taught 'more or less as an aside to the serious business of digesting the classics, and the pupils of various grades and assorted ages took the elementary mathematical course at the convenience of their other studies'.

By the time Galois entered that math class, his grades at the school were poor and his work was suffering from what is best described by Bell as 'acute boredom'; in fact, he had already been made to repeat a year's work. But when he started to read the geometry works of Legendre, Galois lit up. Bell claims that he 'read the geometry cover to cover as easily as other boys read a pirate yarn' and that he felt it to be a true work of art composed by a 'creative genius'. In algebra, Galois had the same enthusiasm for the work of Lagrange, mastering both works in just a single reading.

Galois believed that his great understanding of these works was sufficient to prepare him to enter the École Polytecnique, 'the best school of mathematics in all of France'. His teachers, who had long misunderstood Galois' genius and his uncanny ability to solve problems in his head without working them out on paper, recommended that he remain another term at Louis-le-Grand, in order to improve his 'method'. But Galois' worst enemy foiled him again on the entrance examinations to the Polytecnique - his conceit and his failure to adequately prepare for the exam led to a dismal and crushing failure. Now 17, he remained at Louis-le-Grand for another year, waiting to take the tests for the Polytecnique a second time.

But while he waited, tragedy struck in his young life. His father, Nicholas Galois, 'left home, made the short trip to Paris, and killed himself'. The most tragic part of this event is that the fate of Galois' father was achingly similar to the fate Évariste himself would meet in just three years. Nicholas Galois was the product of a tragic time in France. During his lifetime, Napoleon had ruled France, only to be exiled and then to triumphantly return again. The Galois family remained ardent supporters of the French Republic throughout the whole revolution, and the opposing Jesuit faction did much to undermine political leaders of the Republic, such as Mayor Galois. The Jesuits circulated such vicious rumors about young Galois' father that the man's reputation 'decreased to the point where he was considered the town crackpot' until he finally committed suicide in despair.

Évariste already mistrusted the political establishment and was a staunch Republican in his own right. However, his father's death pushed him into a 'political paranoia' that would soon cause young Galois more serious trouble.

Only days after his father's death, Galois retook his Polytecnique examinations - and failed them a second time. In fact, with this failure, Galois took from himself any chance of ever entering the École Polytecnique, because, frustrated by the examiner's focus on method above imagination, Galois flung an eraser at the examiner, hitting him squarely on the head.

Despite this defeat, Galois persevered and managed to enter the somewhat less prestigious École Normale in November of 1829. While a student at this school, Galois 'feverishly continued his mathematics research' and wrote three papers devoted to the revolutionary ideas of algebraic equation theory. He then submitted some of the results of his work to the Academy of Sciences in Paris. The Academy, in turn, sent his work off to Cauchy, who they considered to be the premier mathematician in France at the time, to be evaluated. However, Galois later learned that Cauchy knew nothing of his great work. He then speculated (probably correctly) that Cauchy had thrown his paper in the wastecan. Interestingly enough, another notable mathematician of the time, Niels Abel, sent a work on fifth-degree algebraic equations to Cauchy as well, only to have it meet the same fate.

Later, when Galois resubmitted his paper to the Academy, it was, in fact, published, and Galois might have been able to begin somewhat of a promising career in mathematics after he finished his work at the École Normale. But once again, Galois brought about his own defeat.

Angered by the political wavering and inaction by the director of the École Normale, Galois decided to write a blistering poisoned-pen letter to the local paper, the Gazette des Écoles, and was promptly expelled from school.

At this point, Évariste (perhaps wisely) decided to abandon his study of mathematics for a time. However, his 'second choice' caused him even more trouble – because Galois devoted himself to revolutionary politics.

According to Engheta, Galois then took it upon himself to join the National Guard, a republican party, and was 'in and out of prison' for several years. His most notable escapade was an ill-fated toast proposed at a Republican party on May 9, 1831, where Galois said he raised his glass to King Louis-Phillipe. But at the same time, Galois held an open knife in his other hand, and his monarchy-hating companions took his words as a threat to the king and began dancing and cheering in the streets. Galois was arrested the next day on charges of treason against the king for inciting the mob and held until June 15 at Sainte-Pélagre prison, but was actually acquitted quickly when his lawyer told the jury that Galois had actually said the toast was 'To King Louis-Phillipe, if he turns traitor.' The jury, sympathetic to the cause of the National Guard, freed Galois the next day.

Galois' freedom did not last, however. On July 14 of 1931, Bastille Day, Galois was arrested once again, this time on the trumped-up charge of carrying illegal weapons and wearing the uniform of the Artillery Guard, which had been disbanded a year earlier. For this, he again landed in prison.

During this time in jail, Galois finally received a reply from the Academy of Sciences on his latest set of papers – but not the reply he had been hoping for. When Poisson, one of the chairs of the academy, read the work, he declared it 'incomprehensible'. In fact, in the letter sent to Galois with the returned paper, the Academy stated:

> It is not even possible for us to give an idea of this paper. M. Galois' proofs are neither sufficiently clear nor sufficiently developed to allow us to judge.

The Academy did encourage Galois to continue on with a more complete account of his work, but that fact was lost on the young man, who then tried to commit suicide by stabbing himself with a dagger. He was prevented from doing so by the other prisoners, but became increasingly fatalistic in his correspondence with friends after the incident.

At the same time, Galois embarked upon a tragic love affair with a woman named Stephanie-Felice du Motel, the daughter of the prison's resident physician. It seems from correspondence that Stephanie tried to remove herself from her affair with Galois, and he as well was despondent over his involvement with her, for in a letter to his friend Raspail, Galois referred to Stephanie as a 'low class coquette'.

In another letter, dated May 25, 1832, Galois wrote to his friend Auguste Chevalier that he was 'disillusioned of everything, even love and fame.' Four days after that, Galois was once again freed.

Little is known of May 29, 1832, the fateful eve preceding the day of Galois' death, or of the events that prompted the challenge that would kill 20-year-old Évariste. Many writers assume that the honor of Stephanie du Motel was at stake, and other authors propose that the duel was of a political nature, a challenge against the Republic that Galois so staunchly defended. Either way, Galois did not wish to fight, but could not avoid it, and it seems from all accounts that he was sure he would lose.

As evening set in on the 29th of May that year, Galois frantically began to write. He scribbled notes upon notes, addressing them to his friend Chevalier, scratching out whole paragraphs once he had begun because he had no time to finish the proofs necessary - and even scrawling the phrase je n'ai pas le temps, 'I have no time' repeatedly in his margins .

Dawn came on May 30, and Évariste had not yet slept. But he rode off to his duel, leaving 60 pages of work and the postscript, 'Later there will be, I hope, some people who will find it to their advantage to decipher all this mess' behind in the care of his friend Chevalier and his younger brother, Alfred.

According to Muir, 'a few hours later, a peasant found him shot through the stomach, alone, and lying in a ditch where his seconds had deserted him'. Galois was taken to a hospital, but refused to be blessed by a priest. His brother Alfred came to his bedside, and soon burst into tears at the sight of his brother. Galois' response to his brother is legendary: 'Don't cry,' he whispered. 'I need all my courage to die at twenty.' And then his life ended, and he was buried in the common ditch of a local cemetery, so that today there remains no trace of his grave.

The works of variste Galois were later published, but not until they were read by the French mathematician and editor Liouville in 1843. In fact, Galois' writings did not appear in print until 1846, and were not widely read until the mid-1850s. But by the end of the nineteenth century, 'Galois was considered to be one of the greatest mathematicians of his time'.

His work revolutionized the study of equations. Among other things, Galois succeeded in proving that equations above the fourth degree cannot be solved generally by radicals, as lower-degree equations can. He also went on to show what the necessary conditions are for such a solution to exist at all.

Galois' most important work came in the theory of groups. He posited three principles for what a 'group' is, and used these groups as a means for comparing types of algebraic equations, which then allowed him to discuss the solutions of these equations. His famous theorem is 'In order that an irreducible equation of prime degree be solvable by radicals, it is necessary and sufficient that all its roots be rational functions of any two of them'.

His three principles defining a group make this theorem and others like it somewhat more understandable. Basically, a group is a set of elements and an operation, say , which may be performed on any two elements a and b of the set S in order to produce a uniquely determined element $a * b$ in S such that the following three postulates are satisfied:

1. The set must be associative, such that $a * (b * c) = (a * b) * c$ for all $a, b, c \in S$.
2. There must exist an identity i in S such that, for all a in S, $a * i = a$.
3. For every element a in S, there must exist an inverse of a, denoted a^{-1}, such that $a * a^{-1} = i$, where i is inverse that was defined in the second postulate.

From these postulates, more properties of groups can be developed, and those properties can be used to compare groups with other groups that seem to be unrelated through a concept called isomorphism, whereby there exists a one-to-one map of the elements of one group onto the elements of another group, such that, for x and y in S, $*'$ a binary operation on S and $*$ a binary operation on

S', $f(x * y) = f(x) *' f(y)$. If there is such a map between the two sets S and S', then those sets are isomorphic to each other, and if this is the case, their properties are the same. This was how Galois proved that no solutions of the general quintic could be found using radicals - he took permutations, or rearrangements, of the possibilities for radical roots in the quintic, which are a group. Then, he compared the structure of that group to groups which were easier to work with, in order to show that no general solution existed. Before that can be done, several general properties of groups must be developed.

Perhaps this theory seems random or abstract. That is exactly what Galois intended. His new mathematics, his 'abstract' algebra, was a foundation for a new way of thinking, a look at the underlying structure and form of mathematics. It was Galois and his incredible genius that brought mathematics out of computation and into logical consideration.

The nature of this eccentric and brilliant man is perhaps best summed up by Évariste Galois himself. A few days before he died, Muir says, he wrote, 'There are those who are destined to do good but never experience it. I believe I am one of them'. And so he was.

In April 1829 Galois had his first mathematics paper published on continued fractions in the Annales de mathématiques. On 25 May and 1 June he submitted articles on the algebraic solution of equations to the Académie des Sciences. Cauchy was appointed as referee of Galois' paper.

Tragedy was to strike Galois for on 2 July 1829 his father committed suicide. The priest of Bourg-la-Reine forged Mayor Galois' name on malicious forged epigrams directed at Galois' own relatives. Galois' father was a good natured man and the scandal that ensued was more than he could stand. He hanged himself in his Paris apartment only a few steps from Louis-le-Grand where his son was studying. Galois was deeply affected by his father's death and it greatly influenced the direction his life was to take.

A few weeks after his father's death, Galois presented himself for examination for entry to the École Polytechnique for the second time. For the second time he failed, perhaps partly because he took it under the worst possible circumstances so soon after his father's death, partly because he was never good at communicating his deep mathematical ideas. Galois therefore resigned himself to enter the École Normale, which was an annex to Louis-le-Grand, and to do so he had to take his Baccalaureate examinations, something he could have avoided by entering the École Polytechnique.

He passed, receiving his degree on 29 December 1829. His examiner in mathematics reported:

> This pupil is sometimes obscure in expressing his ideas, but he is intelligent and shows a remarkable spirit of research.

His literature examiner reported:

> This is the only student who has answered me poorly, he knows absolutely nothing. I was told that this student has an extraordinary capacity for mathematics. This astonishes me greatly, for, after his examination, I believed him to have but little intelligence.

Galois sent Cauchy further work on the theory of equations, but then learned from Bulletin de Frussac of a posthumous article by Abel which overlapped with a part of his work. Galois then took Cauchy's advice and submitted a new article On the condition that an equation be soluble by radicals in February 1830. The paper was sent to Fourier, the secretary of the Academy, to be considered for the Grand Prize in mathematics. Fourier died in April 1830 and Galois' paper was never subsequently found and so never considered for the prize.

Galois, after reading Abel and Jacobi's work, worked on the theory of elliptic functions and abelian integrals. With support from Jacques Sturm, he published three papers in Bulletin de Férussac in April 1830. However, he learnt in June that the prize of the Academy would be awarded the Prize jointly to Abel (posthumously) and to Jacobi, his own work never having been considered.

July 1830 saw a revolution. Charles 10th fled France. There was rioting in the streets of Paris and the director of École Normale, M. Guigniault, locked the students in to avoid them taking part. Galois tried to scale the wall to join the rioting but failed. In December 1830 M. Guigniault wrote newspaper articles attacking the students and Galois wrote a reply in the Gazette des Écoles, attacking M. Guigniault for his actions in locking the students into the school. For this letter Galois was expelled and he joined the Artillery of the National Guard, a Republican branch of the militia. On 31 December 1830 the Artillery of the National Guard was abolished by Royal Decree since the new King Louis-Phillipe felt it was a threat to the throne.

Two minor publications, an abstract in Annales de Gergonne (December 1830) and a letter on the teaching of science in the Gazette des Écoles (2 January 1831) were the last publications during his life. In January 1831 Galois attempted to return to mathematics. He organised some mathematics classes in higher algebra which attracted 40 students to the first meeting but after that the numbers quickly fell off. Galois was invited by Poisson to submit a third version of his memoir on equation to the Academy and he did so on 17 January.

On 18 April Sophie Germain wrote a letter to her friend the mathematician Libri which describes Galois' situation.

> .. the death of M. Fourier, have been too much for this student Galois who, in spite of his impertinence, showed signs of a clever disposition. All this has done so much that he has been expelled form École Normale. He is without money... . They say he will go completely mad. I fear this is true.

Late in 1830 19 officers from the Artillery of the National Guard were arrested and charged with conspiracy to overthrow the government. They were acquitted and on 9 May 1831 200 republicans gathered for a dinner to celebrate the acquittal. During the dinner Galois raised his glass and with an open dagger in his hand appeared to make threats against the King, Louis-Phillipe. After the dinner Galois was arrested and held in Sainte-Pélagie prison. At his trial on 15 June his defense lawyer claimed that Galois had said

> To Louis-Phillipe, if he betrays

but the last words had been drowned by the noise. Galois, rather surprisingly since he essentially repeated the threat from the dock, was acquitted.

The 14th July was Bastille Day and Galois was arrested again. He was wearing the uniform of the Artillery of the National Guard, which was illegal. He was also carrying a loaded rifle, several pistols and a dagger. Galois was sent back to Sainte-Pélagie prison. While in prison he received a rejection of his memoir. Poisson had reported that:

> His argument is neither sufficiently clear nor sufficiently developed to allow us to judge its rigour.

He did, however, encourage Galois to publish a more complete account of his work. While in Sainte-Pélagie prison Galois attempted to commit suicide by stabbing himself with a dagger but the other prisoners prevented him. While drunk in prison he poured out his soul

> Do you know what I lack my friend? I confide it only to you: it is someone whom I can love and love only in spirit. I have lost my father and no one has ever replaced him, do you hear me...?

In March 1832 a cholera epidemic swept Paris and prisoners, including Galois, were transferred to the pension Sieur Faultrier. There he apparently fell in love with Stephanie-Felice du Motel, the daughter of the resident physician. After he was released on 29 April Galois exchanged letters with Stephanie, and it is clear that she tried to distance herself from the affair.

The name Stephanie appears several times as a marginal note in one of Galois' manuscripts.

Galois fought a duel with Perscheux d'Herbinville on 30 May, the reason for the duel not being clear but certainly linked with Stephanie.

The note reads

> There is something to complete in this demonstration. I do not have the time.

It is this which has led to the legend that he spent his last night writing out all he knew about group theory. This story appears to have been exaggerated.

Galois was wounded in the duel and was abandoned by d'Herbinville and his own seconds and found by a peasant. He died in Cochin hospital on 31 May and his funeral was held on 2 June. It was the focus for a Republican rally and riots followed which lasted for several days.

Galois' brother and his friend Chevalier copied his mathematical papers and sent them to Gauss, Jacobi and others. It had been Galois' wish that Jacobi and Gauss should give their opinions on his work. No record exists of any comment these men made. However the papers reached Liouville who, in September 1843, announced to the Academy that he had found in Galois' papers a concise solution

> ...as correct as it is deep of this lovely problem: Given an irreducible equation of prime degree, decide whether or not it is soluble by radicals.

Liouville published these papers of Galois in his Journal in 1846.