Math 307, Fall 2010
Homework 10, due Friday, December 3

Note: In problems (2) and (4), some kind of pictorial representation of the triangulation is all that is required.

(1) Find the fundamental group of the spaces from problem 4 on Homework 9.

(2) (p.124, 1) Find triangulations of the cylinder, the Klein bottle, and the double torus.

(3) (p.124, 5) If X and Y are triangulable spaces, show that $X \times Y$ is triangulable. Please write out all the details of your proof.

(4) (p.124, 8) Show that the dunce hat (Figure 5.11) is triangulable, but that the comb space (Figure 5.10) is not.

(5) (a) (p.131, 14) Use the simplicial approximation theorem to show that if $k < m$ and $k < n$, any map from S^k to S^m is nullhomotopic, and that the same is true of any map from S^k to $S^m \times S^n$. (Hint: A map from a k-simplex to an n-simplex with $k < n$ cannot be onto.)
(b) (p.131, 13) Use part (a) to show that the n-sphere is simply connected for $n \geq 2$.

(6) Which surface in the classification theorem is homeomorphic to the Klein bottle?

(7) (p.152, 3) The connected sum of two surfaces is defined as follows: Remove a disk from each surface and connect up the resulting boundary circles by a cylinder. Assuming this is a well-defined operation, show that the connected sum of a torus with itself is a sphere with two handles, and the connected sum of a projective plane with itself is a Klein bottle.

(8) Show that the torus is orientable and the Klein bottle is not.

(9) (p.158, 8) Let G be a finite group which acts a a group of homeomorphisms of a closed surface S in such a way that the only element with any fixed points is the identity.
(a) Show that the orbit space S/G is a closed surface.
(b) Show that S may be orientable, yet S/G nonorientable.
(c) If S/G is orientable, does S have to be?
(A group action for which only the identity element has any fixed points is said to be *fixed-point free*, and the group is said to *act freely*.)