Math 307, Fall 2008
Homework 5, due Friday, October 15

(1) Let A and B be proper subsets of X and Y, respectively. Show that if X and Y are connected then $(X \times Y) - (A \times B)$ is connected as well.

(2) (p.60, 30) Show that the set of points in $\mathbb{R}^2$ with at least one rational coordinate is connected.

(3) Recall that a space X is *totally disconnected* if the only connected subsets of X are singletons. Show that $\mathbb{R}_l$ is totally disconnected (recall that $\mathbb{R}_l$ is the set of real numbers with the lower-limit topology generated by intervals $[a, b)$).

(4) (p.63, 37) Show that the continuous image of a path-connected space is path-connected.

(5) (p.72, 1) Show that the following identification spaces are homeomorphic (and give $\mathbb{RP}^n$ by definition from class).
  (a) $S^n/\{\text{antipodal points}\}$
  (b) $(\mathbb{R}^{n+1} - \{0\})/\{\text{lines through origin}\}$

(6) Let D^2 be the unit disk in $\mathbb{R}^2$ and let M be the Möbius band. Show that if $H : \partial D^2 \to \partial M$ is a homeomorphism then $D^2 \cup_H M \cong \mathbb{RP}^2$. A sequence of pictures will do.

(7) (p.72, 2) What is the result when the boundary circle of the Möbius band is identified to a point? Explanation with pictures is fine.

(8) (p.72, 7) Describe each of the following spaces (i.e. give a more familiar description of what they are).
  (a) The cylinder with each of its boundary circles identified to a point;
  (b) The torus with the a meridional and a longitudinal circle identified to a point;
  (c) S^2 with the equator identified to a point;
  (d) $\mathbb{R}^2$ with each of the circles centered at the origin and of integer radius identified to a point.