Math 307, Fall 2010

Homework 9, due Friday, November 19

(1) Recall that we defined space X to be *contractible* in two ways (once in an earlier homework and once in class):
   Definition 1: X is contractible if it is is homotopy equivalent to a point; and
   Definition 2: X is contractible if the identity map of X is nullhomotopic.
Show these two definitions are equivalent.

(2) (p.109, 24)
   (a) Show that if $X \simeq Y$ and $X' \simeq Y'$ then $X \times X' \simeq Y \times Y'$.
   (b) Show that, for any space X, CX (cone on X) is contractible.

(3) (p.109, 29) Show with pictures that the house with two rooms is contractible (see Figure 5.12 on page 109 in Armstrong).

(4) Find the homotopy type of the following spaces. In other words, find a more familiar space which is homotopy equivalent to the given space. You may do this with pictures.
   (a) $T^2 - \{\text{point}\}$
   (b) $\mathbb{R}^2 - \{2 \text{ points}\}$
   (c) $S^1 \cup \{(x, 0) \mid x \in (-1, 1)\}$
   (d) $T^2 - \{\text{non-seperating closed curve}\}$
   (e) $S^2 - \{3 \text{ points}\}$