Math 349 Algebraic Geometry, Spring 2009
Homework 10, due Friday, May 1

(1) Prove the other half of Theorem 9 in Chapter 5, §4 (one half is proven right before the statement of the theorem).

(2) Suppose A is an invertible $n \times n$ matrix over a field k. Consider the morphism of varieties

$$\begin{aligned} L_A \colon k^n &\longrightarrow k^n \\ x &\longmapsto Ax. \end{aligned}$$

(Here x is thought of as a column vector.) Show that the corresponding k-algebra homomorphism $L_A^* \colon k[x_1, ..., x_n] \to k[x_1, ..., x_n]$ is an isomorphism and deduce that L_A is an isomorphism of varieties.

(3) Chapter 5, §4, problem 4

(4) Chapter 5, §4, problem 5

(5) Chapter 5, §4, problem 6

(6) Chapter 5, §4, problem 13

(7) Show that the Zariski topology on k^n is really a topology.

(8) Show that a presheaf on a space X can be thought of as a contravariant functor from $\mathcal{O}(X)$, the category of open subset of X (what are the morphisms in this category?) to some category $\mathcal{C}$.