Math 349 Algebraic Geometry, Spring 2009
Homework 2, due Friday, February 20

Note that problems in this assignment jump around a bit with respect to sections of the book, but this is consistent with how the material was presented in class.

(1) It turns out that in this class we'll mostly be concerned with polynomials over infinite fields since finite fields exhibit pathologies which make them hard to work with. For example, one can have non-zero polynomials with coefficients in $\mathbb{F}_p$, where p is a prime, which vanish on every element on $\mathbb{F}_p$. We don't like this. Exhibit such a polynomial for an arbitrary prime p. (Hint: You probably did this in your first-semester abstract algebra class; use Lagrange's Theorem.)

(2) §4, problem 2

(3) §4, problem 3(b)

(4) §4, problem 4

(5) Given $f(x) = 2x^7 + x^3 - 1$ and $g(x) = x^3 + 4$ in $\mathbb{Q}[x]$, find
 (a) the quotient and remainder upon dividing f by g;
 (b) the generator of the ideal $\langle f, g \rangle$.
 (For the second part, you might want to remind yourselves of the Euclidean Algorithm.)

(6) §5, problem 3

(7) §2, problem 1(b)(c)

(8) §2, problem 8