Math 349 Algebraic Geometry, Spring 2009
Homework 4, due Friday, March 6

(1) Chapter 2, §4, problem 9

(2) (a) Show that the ring of Gaussian integers $\mathbb{Z}[i] = \{a + bi \mid a, b \in \mathbb{Z}, i^2 = -1\}$ is Noetherian. (Hint: Look up the definition of a Euclidean Domain and show $\mathbb{Z}[i]$ is one. Then use the fact (you do not need to prove it) that every Euclidean Domain is a PID.)
    (b) Suppose that R is a Noetherian ring and I is an ideal of R. Show that R/I is also Noetherian.

(3) Show that the following rings are not Noetherian.
    (a) The ring $k[x_1, x_2, x_3, ...]$ of polynomials in infinitely many variables over a field k.
    (b) The ring of continuous functions from $\mathbb{R}$ to $\mathbb{R}$.

(4) Chapter 2, §5, problem 7

(5) Chapter 2, §5, problem 9

(6) Chapter 2, §5, problem 16

(7) Chapter 2, §6, problem 3

(8) Solve the system of equations $2x^2 + 2xy + y^2 - 2x - 2y = 0$ and $x^2 + y^2 = 1$ over $\mathbb{Q}$ by finding the Groebner basis for an appropriate ideal on `http://www.geocities.com/famancin/groebner_applet.html` with the lex order $x > y$. One of the polynomials in your basis should be in terms of y only. Now repeat the problem but with the lex order $y > x$. Now one of the polynomials in your basis should be in terms of x only. You should of course get the same set of solutions. Keep in mind that you can always multiply elements of your basis by constants to simplify them.