

Math 349 Algebraic Geometry, Spring 2009
Homework 5, due Friday, March 13

(1) Chapter 2, §7, problem 2(a) (but just for lex order)

(2) Prove one direction of the Hilbert Nullstellensatz:

If $f^m \in \langle f_1, f_2, \dots, f_s \rangle$ for some $m \geq 1$, then $f \in \mathbb{I}(\mathbb{V}(f_1, f_2, \dots, f_s))$.

(3) Chapter 4, §1, problem 4

(4) Chapter 4, §1, problem 5

(5) Chapter 4, §2, problem 2

(6) Chapter 4, §2, problem 4(b)(c)

(7) Chapter 4, §2, problem 7(a)

(8) Show that the strong Nullstellensatz does not necessarily hold for fields that are not algebraically closed by considering the ideal $I = \langle x^{2n} + y^{2n} + 1 \rangle \in \mathbb{R}[x, y]$, $n \in \mathbb{N}$, and the composition $\mathbb{I}(\mathbb{V}(I))$.