Math 349 Algebraic Geometry, Spring 2009
Homework 6, due Friday, March 20

(1) Let R be a ring and I, J ideals in R.
  (a) Prove that IJ is an ideal contained in $I \cap J$.
  (b) Give an example where $IJ \neq I \cap J$.
  (c) Prove that, if R is commutative and $I + J = R$, then $IJ = I \cap J$.

(2) Chapter 4, §3, problem 6

(3) Chapter 4, §3, problem 9

(4) Chapter 4, §4, problem 1

(5) Chapter 4, §4, problem 5

(6) Chapter 4, §4, problem 8(a)-(c)

(7) Suppose R is a commutative ring.
  (a) Show M is a maximal ideal of R if and only if the quotient ring R/M is a field.
  (b) Show P is a prime ideal of R if and only if the quotient ring R/P is an integral domain.
  (c) Use parts (a) and (b) to show that every maximal ideal of R is prime.

(8) Let R be the ring of continuous functions from $[0, 1]$ to $\mathbb{R}$. Given $a \in R$, consider the function

$$\begin{aligned} ev_a \colon R &\longrightarrow \mathbb{R} \\ f &\longmapsto f(a). \end{aligned}$$

Show that the kernel of this function is a maximal ideal of R.