

① Show that the Vassiliev skein relation is well-defined with respect to the order of crossings resolved:

$$\begin{aligned}
 V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) &= V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) \\
 &\quad \downarrow \text{resolve this first} \quad - \quad V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) \\
 &= V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) - V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) \\
 &- V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) + V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right)
 \end{aligned}$$

On the other hand,

$$\begin{aligned}
 V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) &= V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) \\
 &\quad \downarrow \text{resolve this first} \quad - \quad V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) \\
 &= V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) - V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) \\
 &- V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right) + V\left(\begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array} \cdots \begin{array}{c} \nearrow \searrow \\ \times \\ \searrow \nearrow \end{array}\right)
 \end{aligned}$$

But the four terms are the same,
with same signs.

Since the order in which 2 singularities are
resolved doesn't matter, by induction the order
in which n singularities are resolved does
not matter.

2) Show only type 1 invariants are constant
functions on K .

V type 1 \Rightarrow ~~value of V on an 1-singularity~~
value of V on an 1-singularity
but only depends on the
placement of singularity. \textcircled{X}

So

$$V(\underbrace{\nearrow \searrow}_{\substack{\text{vassiliev} \\ \text{slab-} \\ \text{rel'n}}}) = V(\underbrace{\searrow \nearrow}_{\text{Some 1-singularity but}}) + V(\searrow \nearrow)$$

$$= V(\infty) + V(\searrow \nearrow)$$

choose a particular 1-singularity
but since V doesn't see
the difference
by \textcircled{D}

$$\begin{aligned}
&= v(\infty) - v(\infty) + v(\lambda^{\uparrow}) \quad (3) \\
&= v(0) - v(0) + v(\lambda^{\uparrow}) \\
&\quad \swarrow \quad \searrow \\
&\quad \text{invariants} \\
&= v(\lambda^{\uparrow})
\end{aligned}$$

Since $v(\text{root}) = v(\text{root w/ crossing changed})$,

$$v = \text{constant.}$$

(Same as proof in class that only type 0 invariants are constant)