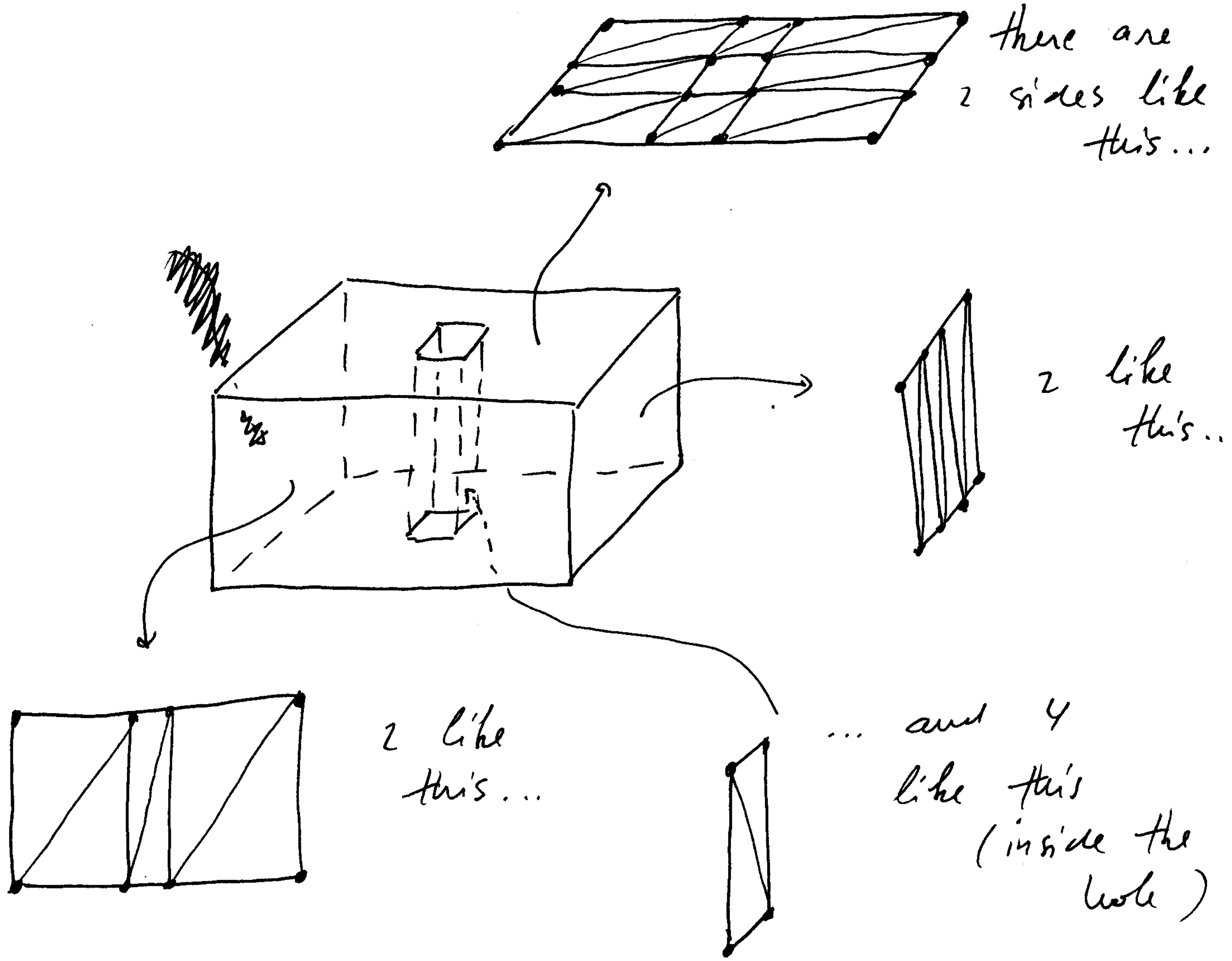
HOMEWORK 3 SOLUTIONS

(1) Triangulate the torus



- Note that lots of vertices and edges are repeated.
- This is not a very efficient triangulation. Using squares as 2-cells would have been better.

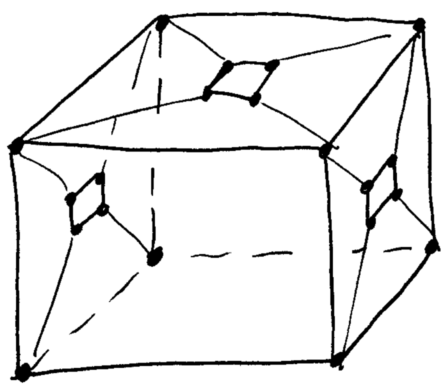
So we get

$$v = 32$$
$$e = 96$$
$$f = 64$$

$$\Rightarrow \chi(T^2) = 0$$

2 <u>Exercise 4.10, page 83</u>

Let's triangulate the "outer shell" first:



$$\left.\begin{array}{l} v = 20 \\ e = 36 \\ f = 15 \end{array}\right\} \chi = -1$$

(using general 2-cells, like  and )

Now notice that "inside box" looks exactly like the outer shell, so Euler characteristic picks up another -1.

But that's it!

There are no vertices left after we triangulate outer shell and inner box. To triangulate the rest, we would draw a bunch of edges but it is not hard to see that there would be as many extra edges we didn't count as there are faces we didn't count. These of course cancel in the Euler characteristic (I can elaborate on this ~~...~~ in class or in office hours).

So $\chi(\text{surface}) = -2 = 2-2g$

$\Rightarrow \boxed{g=2}$

It is also not hard to see a cut-and-paste homeomorphism between the surface that's given and the standard double torus.