

HW 5 SOLUTIONSExercise 6.2

$$\begin{aligned}
 \langle \text{Diagram 1} \rangle &= A \langle \text{Diagram 2} \rangle + A^{-1} \langle \text{Diagram 3} \rangle \\
 &= A \left[A \langle \text{Diagram 4} \rangle + A^{-1} \langle \text{Diagram 5} \rangle \right] \\
 &\quad + A^{-1} \left[A \langle \text{Diagram 6} \rangle + A^{-1} \langle \text{Diagram 7} \rangle \right]
 \end{aligned}$$

$$= A^2 \langle \infty \circ \rangle + \langle \infty \rangle + \langle \infty \rangle + A^{-2} \langle \text{Diagram 8} \rangle$$

$$= A^2(-A^2 - A^{-2}) \langle \infty \rangle + 2 \langle \infty \rangle + A^{-2}(-A^{-3}) \langle \text{Diagram 8} \rangle$$

$$= (-A^4 - 1 + 2) \langle \infty \rangle - A^{-5} \langle \text{Diagram 9} \rangle$$

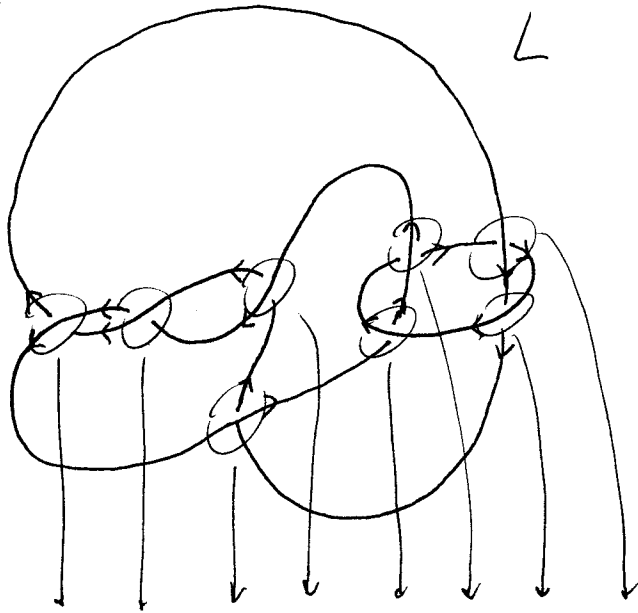
$$= (-A^4 + 1)(-A^{\overset{3}{\cancel{4}}}) \langle \text{Diagram 9} \rangle - A^{-5}$$

$$= (-A^4 + 1)(-A^{\overset{3}{\cancel{4}}}) - A^{-5}$$

$$= A^7 - A^3 - A^{-5}$$

Exercise 6.3

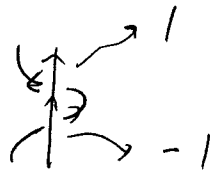
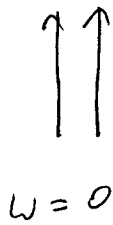
②



$$w(L) = -1 -1 +1 -1 -1 -1 +1 +1 = -2$$

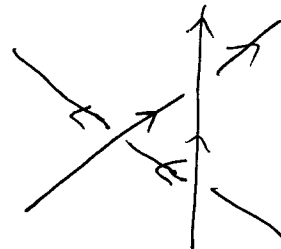
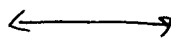
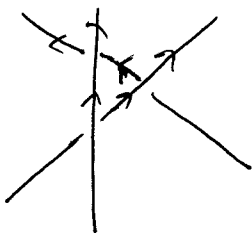
Exercise 6.4

Type II:



$$w = 1 - 1 = 0$$

Type III:



Can match crossings on the left and right, so we change in writhe.

Exercise 6.5:

(3)

From exercise 6.2, we know

$$\langle \mathcal{G} \rangle = A^7 - A^3 - A^{-5},$$

and $w(\mathcal{G}) = -3$, so

$$\begin{aligned} X(\mathcal{G}) &= (-A^3)^{-(-3)} (A^7 - A^3 - A^{-5}) \\ &= -A^{16} + A^{12} + A^4 \end{aligned}$$

Since writhe of a link doesn't change if its orientation changes, and since $\langle \cdot \rangle$ never depends on the orientation, $X(\mathcal{G})$ is the same for either orientation.

On the other hand,

$$w(\mathcal{G}^{\rightarrow}) = 2 \quad \text{but} \quad w(\mathcal{G}^{\leftarrow}) = -2, \text{ so}$$

$$X(\mathcal{G}^{\rightarrow}) = (-A^3)^2 (-A^4 - A^{-4}) = +A^{10} + A^2$$

$$\text{but} \quad X(\mathcal{G}^{\leftarrow}) = (-A^3)^{-2} (-A^4 - A^{-4}) = A^{-2} + A^{-10}$$

Nothing changes if orientation on both components is changed.