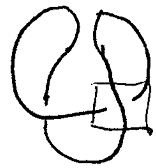
HW 5 SOLUTIONS

Exercise 6.2

$\langle$  $\rangle = A\langle$  $\rangle + A^{-1}\langle$  $\rangle$

$= A\big[A\langle$  $\rangle + A^{-1}\langle$  $\rangle\big]$

$+ A^{-1}\big[A\langle$  $\rangle + A^{-1}\langle$  $\rangle\big]$

$= A^2\langle \infty \ \bigcirc \rangle + \langle \infty \rangle + \langle \infty \rangle + A^{-2}\langle$  $\rangle$

$= A^2(-A^2-A^{-2})\langle \infty \rangle + 2\langle \infty \rangle + A^{-2}(-A^{-3})\langle$  $\rangle$

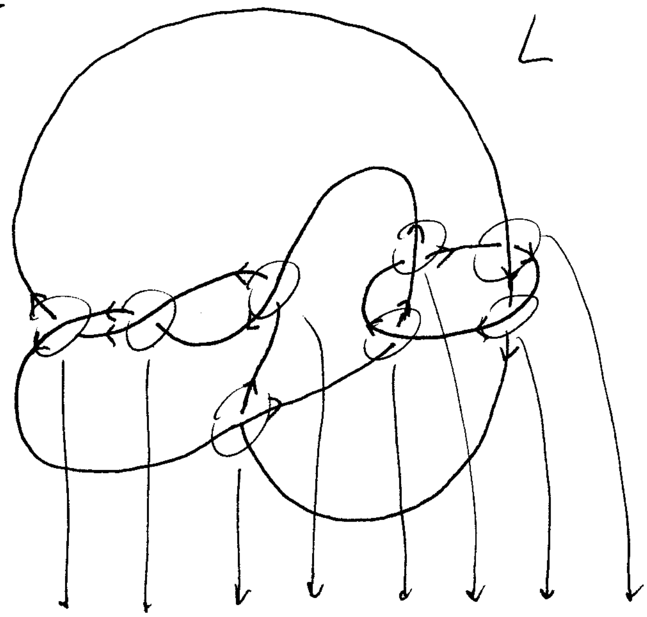
$= (-A^4-1+2)\langle \infty \rangle - A^{-5}\langle \bigcirc \rangle$

$= (-A^4+1)(-A^3)\langle \bigcirc \rangle - A^{-5}$

$= (-A^4+1)(-A^3) - A^{-5}$

$= A^7 - A^3 - A^{-5}$

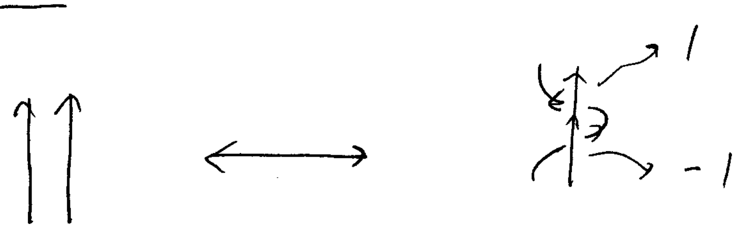
Exercise 6.3



$w(L) = -1 -1 +1 -1 -1 -1 +1 +1 = -2$

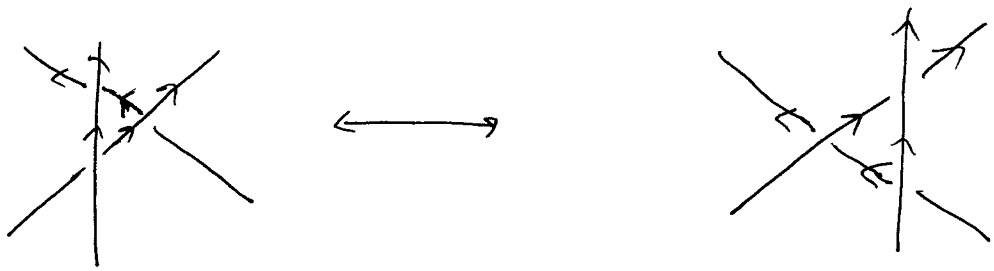
Exercise 6.4

Type II:



$w = 0$ $\qquad$ $w = 1 - 1 = 0$

Type III:



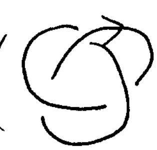
Can match crossings on the left and right, so no change in writhe.

Exercise 6.5:

From exercise 6.2, we know

$\langle$  $\rangle = A^7 - A^3 - A^{-5}$,

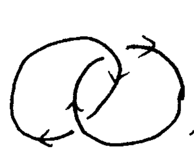
and $w($  $) = -3$, so

$X($  $) = (-A^3)^{-(-3)}(A^7 - A^3 - A^{-5})$

$= -A^{16} + A^{12} + A^4$

Since writhe of a knot doesn't change if its orientation changes, and since $\langle\cdot\rangle$ never depends on the orientation, $X($  $)$ is the same for either orientation.

On the other hand,

$w($  $) = 2$ but $w($  $) = -2$, so

$X($  $) = (-A^3)^2(-A^4 - A^{-4}) = +A^{10} + A^2$

but $X($  $) = (-A^3)^{-2}(-A^4 - A^{-4}) = A^{-2} + A^{-10}$

Nothing changes if orientation on both components is changed.