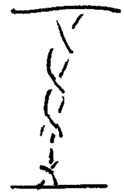
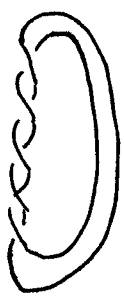
# HW 9 SOLUTIONS

## Exercise 5.17

If the braid  has $k$ crossings,

then $k$ odd gives a knot, e.g.



which is always an $(k,2)$ torus knot.

$k$ odd gives a link with linking # $k/2$, e.g.

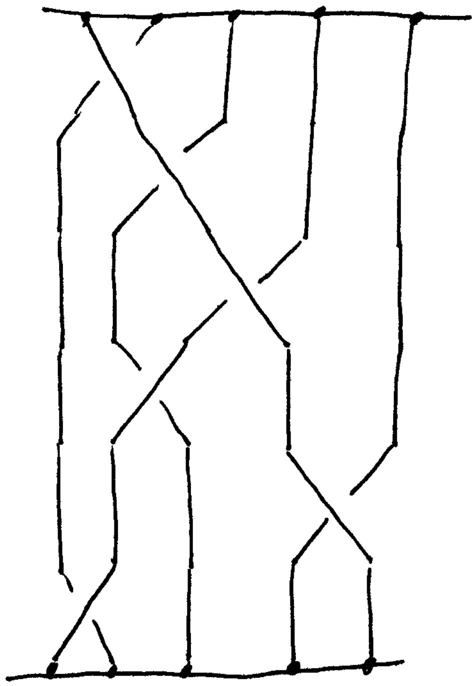
(this is a very special kind of link).

$k=0$ gives unlink of 2 component,

 .

Exercise 5.18  word  $\sigma_1 \sigma_2 \sigma_3 \sigma_2^{-1} \sigma_4 \sigma_1^{-1}$

gives



Exercise 5.21.

The goal is to show that, if $\gcd(m,n) > 1$, some strand will come back to itself through a cycle of strands which does not include all strands. E.g. if $n = 6$ and $m = 4$, one strand cycle is $3 \longrightarrow 6 \longrightarrow 3$, which means that the braid closure will have at least 2 components since the cycle didn't go through all strands.

(Those of you who know about permutation groups should find this very familiar.)

First assume $m < n$ (If $m \geq n$, we can reduce it modulo $n$ which means we disregard the products which are multiples of $n$ since they bring each strand back to itself. For our purposes, this is redundant — think about it!)

Now label the strands $0, 1, \ldots, n-1$ instead of $1, 2, \ldots, n$. This simplifies things since now it is not hard to see that the braid we're looking at satisfies

$$\text{strand } i \longmapsto i+m-1 \ (\text{modulo } n)$$

Similarly,

$$\text{strand } i+m-1 \longmapsto i+2m-2 \ (\text{modulo } n), \text{ etc.}$$

We thus get a cycle, for strand $m$ in particular, which is

$$m,\ 2m-1 \ (\text{mod } n),\ 3m-2 \ (\text{mod } n),\ \ldots,\ \underset{\substack{\parallel \\ m^2-m-1}}{mm-(m-1)} \ (\text{mod } n)$$

Now, if we can show at least one of numbers $0, \ldots, n-1$ <u>does</u> <u>not</u> <u>show</u> <u>up</u> in this list, then the closure of the braid cannot be a knot (<u>think about it!</u>), assuming $m$ and $n$ are <u>not</u> relatively prime. I'll let you think about this (and maybe assign it as a problem on the final!)