Homework 1A: due at the beginning of class Monday Jan. 30

Please read your class notes, pp. 1 - 7 of the introduction, and the worked examples (pp. 17 - 19) in the text.

1. Please read the information sheet handed out on the first day of class. Visit our course Google site.

2. Send me a short email message (a paragraph or two) so I can start to get to know you. Use the subject “HW-1A-Q2” so I can distinguish it from an email that needs an answer. Please include your math history. I am particularly interested in knowing what math you’ve taken recently and hearing about your experience in recent math classes. Other possible topics to include: where you’re from, your interests/hobbies, other classes you are taking this semester, what you hope to get out of this class.

3. Use induction to prove that $\sum_{k=0}^{n} a^k = \frac{1 - a^{n+1}}{1 - a}$ (for $a \neq 1$).

4. (⋆) Prove that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}$ for every integer $n > 1$.

5. Define $\{x_n\}$ as follows: $x_1 = 1$, $x_{n+1} = \frac{1}{1 + x_n}$.

Show by induction that $x_n = \frac{z_n}{z_{n+1}}$, where $\{z_n\}$ is the Fibonacci sequence: $z_1 = z_2 = 1$, $z_{n+2} = z_{n+1} + z_n$.

Homework 1B: due at the beginning of class Thursday Feb. 2

Please read your class notes and Section 1.1 in our textbook.

1. **Well Ordering Principle**: Every non-empty subset of the positive integers contains a least element.

Show that the Well Ordering Principle follows from the Principle of Mathematical Induction.

2. (⋆) Show conversely that the Well Ordering Principle implies the Principle of Mathematical Induction.

*Hint:* Suppose that $\{P_n\}$ is a collection of statements satisfying the conditions for mathematical induction, but suppose that $P_n$ is not true for all integers $n \geq 1$. Let $S$ denote the set of integers for which $P_n$ is false, and obtain a contradiction.