APPLICATIONS HANDOUT

Here are some applications of the linear algebra we've been doing. Work in teams to find answers to these problems.

1. Suppose that two cities — Andyville and Schultzburg — each produces its own special commodity, call them A and S. Suppose also that Andyville consumes 80% of the its own production of good A and 10% of Schultzburg's production of good S; Schultzburg consumes 90% of its own production of good S and 20% of Andyville's production of good A. Find prices p_A and p_S (i.e., dollar values) for the total production of goods A and S so that each city's expenditures match their income.

2. In a particular swamp, let $F(t)$ be the number of fish (in millions) and $C(t)$ be the number of crocodiles (in thousands) in the swamp in year t. Research has shown that the values $F(t+1)$ and $C(t+1)$ can be predicted based on the values of $F(t)$ and $C(t)$:

$$\begin{array}{rcrcl} F(t+1) & = & \frac{11}{10}F(t) & - & \frac{1}{8}C(t) \\ C(t+1) & = & \frac{2}{5}F(t) & + & \frac{1}{2}C(t) \end{array}.$$

Determine values $F(t)$ and $C(t)$ so that $F(t) = F(t+1)$ and $C(t) = C(t+1)$. Can you describe all such 'stable' populations?

3. Silver (Ag) tarnishes when it comes into contact with sulfur, turning nice Ag atoms into nasty atoms of silver sulfide (Ag_2S). Fortunately, sulfur atoms are more strongly attracted to aluminum (Al), and so silver sulfide reacts with aluminum to 'polish' silver objects (removing the tarnish without removing the silver). The chemical reaction that takes place when silver is polished replaces molecules of silver sulfide (Ag_2S) and aluminum (Al) with pure silver (Ag) and aluminum sulfide (Al_2S_3):

$$x_1 Ag_2S + x_2 Al \to x_3 Ag + x_4 Al_2S_3.$$

Balance this chemical equation; i.e., find values of x_1, x_2, x_3 and x_4 so that the number of silver, sulfur and aluminum atoms on the left hand side is the same as the number of corresponding atoms on the right.

4. A trucking company has three warehouses A, B and C. C is the only warehouse which receives product directly from the manufacturer, and it receives 80 units per day. It ships these units to warehouses A and B. A sends products to distributors, and must provide 30 units per day. B also sends units to distributors, but it can send as many or as little units as it likes. For this reason, it sometimes has to send units to warehouse A so it can meet its quota.

The warehouse manager is having a hard time determining how products should move between A, B and C. Your job is to balance the system; find the amount of goods that should be shipped between the warehouses, together with the amount of goods that should be sent from warehouse B to distributors, so that warehouse A meets its quota. Can you describe all ways to balance the system?