Linear algebra in your daily (digital) life

Andrew Schultz

Wellesley College

December 11, 2017

Outline of the talk

- Who cares about eigenvalues?

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    - Google's PageRank algorithm

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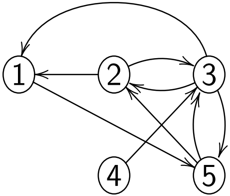
The general constraints

Suppose that G is a directed graph



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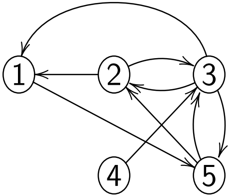

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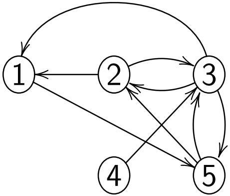

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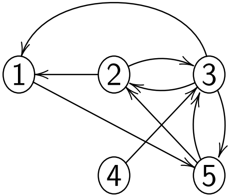

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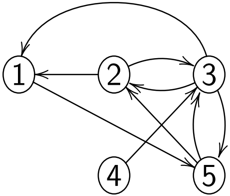

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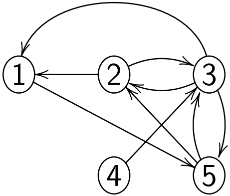

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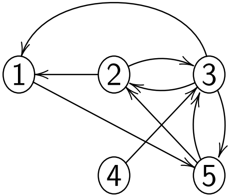

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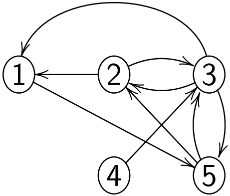

The first approach

$(j) \longrightarrow (i)$ is a vote for (i) from (j)

Form matrix L so that

$$L_{i,j} = \begin{cases} 1 & , \text{ if } (j) \longrightarrow (i) \\ 0 & , \text{ else} \end{cases}$$

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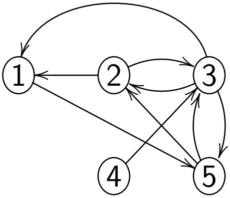

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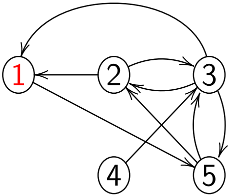

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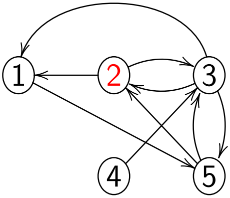

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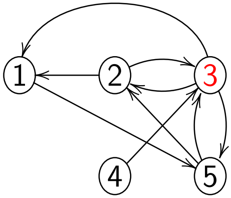

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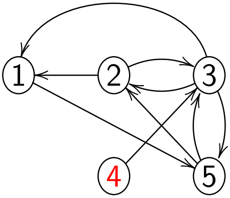

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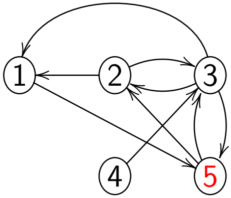

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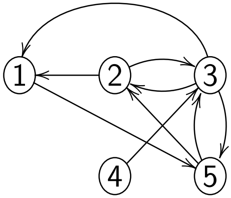

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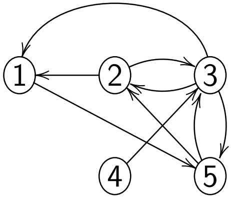
Page 3 wins (score 3), Pages 1,2,5 tie for second (score 2), Page 4 loses



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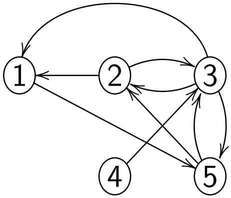

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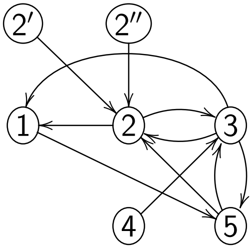

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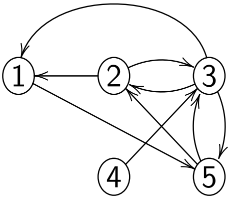
Each page gets a total of 1 vote:

$$\ell(j) = \sum_i L_{i,j}$$



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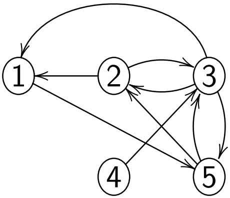

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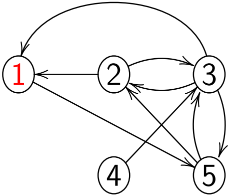

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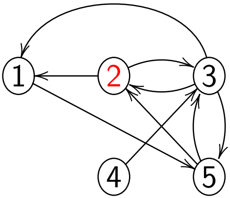

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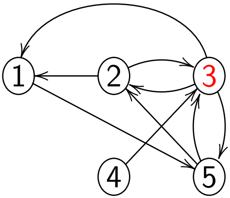

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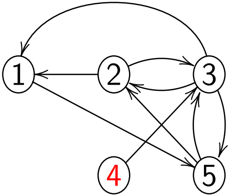

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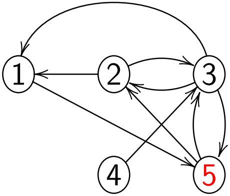

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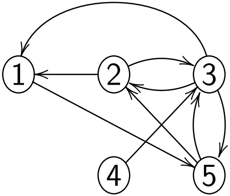

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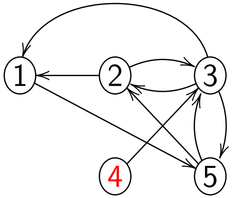

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$$\begin{pmatrix} 0.41 \\ 0.47 \\ 0.52 \\ 0 \\ 0.58 \end{pmatrix}$$ is 1-eigenvector

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Let $PR = dW + (1-d)\begin{pmatrix} \frac{1}{n} & \cdots & \frac{1}{n} \\ \vdots & \ddots & \vdots \\ \frac{1}{n} & \cdots & \frac{1}{n} \end{pmatrix}$

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  - Current age of the universe is about 10^{10} years.
  - If a supercomputer started at the dawn of the universe, then today it would be 0.0000001% done.

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Hence $\lim_{k\to\infty} PR^k\overrightarrow{v}$ is an eigenvalue with eigenvalue 1.

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If $\sum_{i>s} \sigma_i$ is "insignificant," then we have $A \approx \sum_{i=1}^{s} A_i$

Noise Filtering

Matrix representations of images

You can think of an image as a matrix.

- Each pixel contains a gray value
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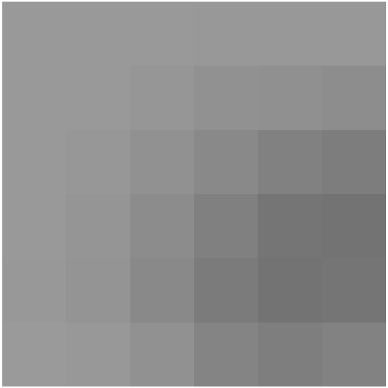

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153	153	153	152	152	152
153	153	150	145	144	141
153	151	145	137	129	125
153	149	140	128	117	115
152	148	137	123	115	117
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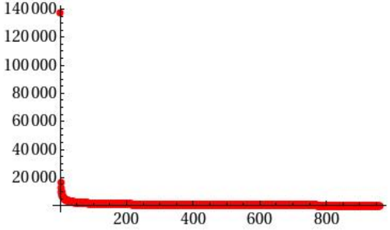
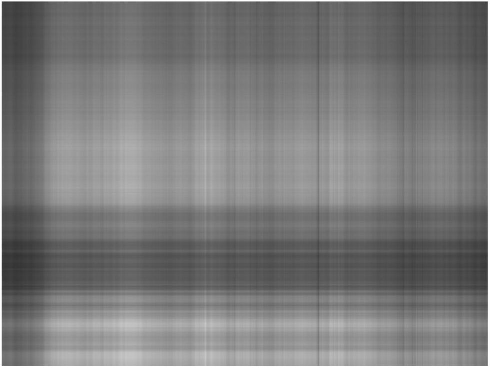


$$\sigma_1 \approx 138,000$$
$$\sigma_2 \approx 17,000$$
$$\sigma_{50} \approx 2,200$$
$$\sigma_{200} \approx 900$$

Some approximations

Let's see what our truncated matrix "looks like"

- s=1
- Compression: 0.18%



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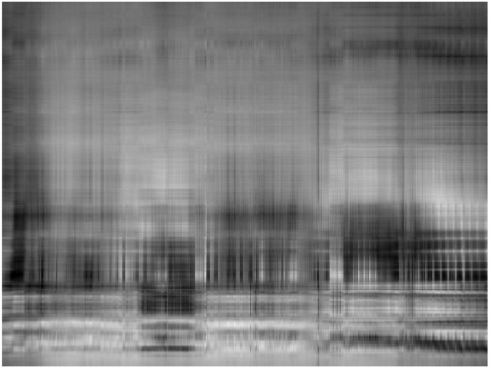
- s=5
- Compression: 0.9%



Some approximations

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- s=10
- Compression: 1.8%

Some approximations

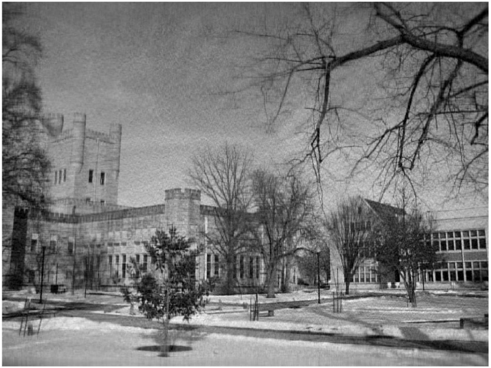
Let's see what our truncated matrix "looks like"

- s=25
- Compression: 4.5%

Some approximations

Let's see what our truncated matrix "looks like"

- s=100
- Compression: 18%



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- takes advantage of properties of images

The “usual” way of thinking about a matrix

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- Each $E(i,j)$ solely responsible for its local behavior.
- Deleting an a_{ij} completely wipes out pixel info.

Rewriting the matrix

What if we chose a different basis for $n \times n$ matrices?

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- deleting $c_{i,j}$ has gradual (though global) effect

A potential basis

We'll choose an basis of $\mathbb{R}^n$ from Fourier series

$$\vec{f}_i = \alpha_i \left\{ \cos\left[\frac{\pi}{n}\left(\frac{2j+1}{2}\right) i\right] \right\}_{j=0}^{n-1}$$

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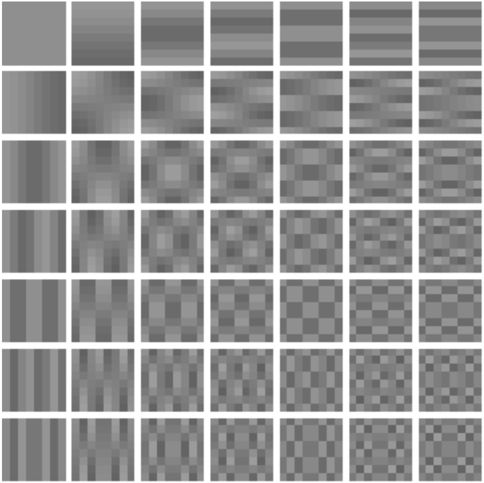
$$B(i,j) = \begin{pmatrix} \vec{f}_0 \\ \hline \vdots \\ \hline \vec{f}_{n-1} \end{pmatrix} E(i,j) \left(\vec{f}_0 \,\middle|\, \cdots \,\middle|\, \vec{f}_{n-1} \right)$$

Seeing the new basis ($n = 8$)



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Then decompression is

- De-quantize
- Change back to standard coordinates
- Reassemble the 8×8 blocks

Working through an example

Extract an 8×8 block

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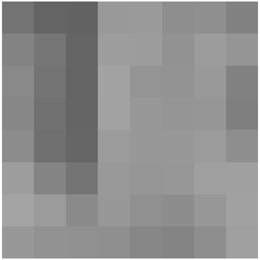
Extract an 8×8 block



$$\longrightarrow \begin{pmatrix} 115 & 100 & 98 & 153 & 154 & 142 & 143 & 130 \\ 131 & 118 & 101 & 157 & 156 & 146 & 156 & 149 \\ 137 & 115 & 100 & 163 & 148 & 147 & 153 & 130 \\ 135 & 113 & 101 & 163 & 152 & 149 & 150 & 127 \\ 140 & 111 & 102 & 156 & 152 & 152 & 155 & 142 \\ 157 & 132 & 116 & 153 & 150 & 151 & 159 & 160 \\ 164 & 155 & 138 & 152 & 144 & 141 & 151 & 161 \\ 152 & 146 & 145 & 143 & 135 & 132 & 142 & 159 \end{pmatrix}$$

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$$\left[\frac{\begin{pmatrix} 1112 & -61 & -14 & 44 & 57 & 34 & -32 & -26 \\ -43 & -36 & -43 & 25 & 13 & 12 & -15 & -8 \\ 2 & 12 & 12 & -26 & -8 & -16 & 7 & 10 \\ 2 & -14 & 1 & 7 & 6 & -3 & 1 & 2 \\ -25 & -4 & -16 & 0 & -1 & 2 & 5 & 3 \\ -2 & 12 & -6 & 1 & -3 & 2 & -1 & -2 \\ -9 & -1 & -2 & 3 & 0 & 5 & 2 & 0 \\ -4 & 2 & -2 & 1 & -1 & 3 & 1 & -1 \end{pmatrix}}{\begin{pmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 71 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{pmatrix}}\right] \longrightarrow$$

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We're down to 16 pieces of information!

Reconstituting our image

Here's the result of reversing this process:

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Original

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Compressed

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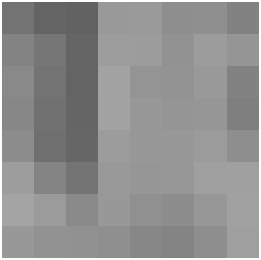
151	132	118	153	155	145	169	153
162	144	129	155	147	135	165	158
166	150	132	152	137	122	157	154

- Average difference $= 5.73$
- Std Dev $= 4.22$

Seeing is believing

Original



Compressed

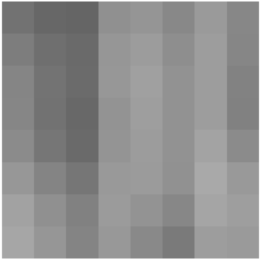


Image Processing

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Note: Here I won't split the image into 8×8 blocks – I want all the information about the image simultaneously

Image Processing

Smooth Part: $B(i,j)$ components for small j, small i

Image Processing

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Image Processing

Horizontal Edges: $B(i, j)$ components for small j, large i

Image Processing

Horizontal Edges: $B(i, j)$ components for small j, large i

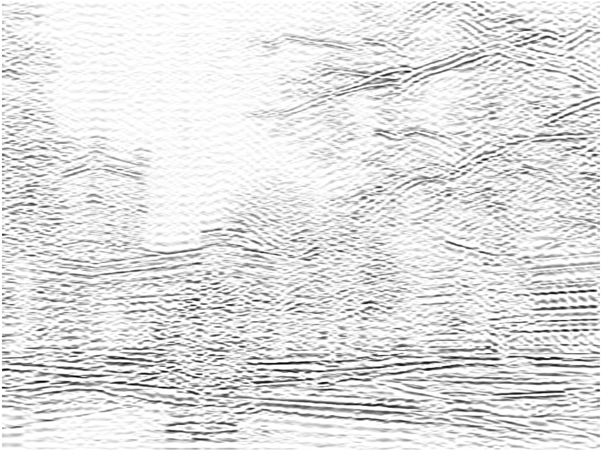


Image Processing

Vertical Edges: $B(i,j)$ components for small i, large j

Image Processing

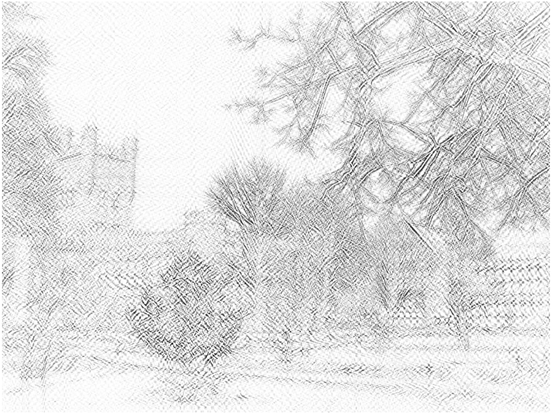
Vertical Edges: $B(i,j)$ components for small i, large j

Image Processing

Scattered Edges: $B(i,j)$ components for large i, large j

Image Processing

Scattered Edges: $B(i,j)$ components for large i, large j



How MP3 compression works

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- Filter out "unnecessary" data using psychoacoustics

Psychoacoustics

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- Temporal Masking - Some weak sounds aren't heard if played right after (or right before!) a louder sound
- Hass effect - If the same tone hits one ear just before another, then your brain perceives it as coming only from the first direction

Thanks!

Thank you!